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Do Volatility Smiles Matter for Pricing Asian Basket Options? The Case of Livestock Gross Margin Insurance for Dairy Cattle

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ABSTRACT

Livestock Gross Margin Insurance for Dairy Cattle (LGM-Dairy) is Asian basket option-like insurance tool that enables U.S. dairy producers to protect income-over-feed-costs margins. While LGM-Dairy rating method assumes flat implied volatility curves for all marginal price distributions, substantial evidence suggests upward-bending skews in implied volatility curves are typical and expected features of options written on futures for storable commodities. While volatility smiles and skews influence premiums considerably when there is only one source of risk, the basket option nature of LGM-Dairy suffices to make the lognormality assumption a useful heuristic that does not bias LGM-Dairy premiums in a financially important way.

Keywords: Revenue Insurance, Skewness, Kurtosis, Volatility, Generalized Lambda Distribution, Crop Insurance, LGM-Dairy

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Do Volatility Smiles Matter for Pricing Asian Basket Options? The Case of Livestock Gross Margin Insurance for Dairy Cattle

Executive Summary:

Livestock Gross Margin Insurance for Dairy Cattle (LGM-D) is an insurance tool that enables dairy producers to protect income-over-feed-cost margins. LGM-Dairy is priced based on information implied from Chicago Mercantile Exchange (CME) futures and options prices for class III milk, corn and soybean meal. The LGM-Dairy rating methodology assumes that the volatility implied from CME futures and options data does not change across strike prices. However, evidence suggests that the volatility surface is not flat. Storable commodities such as corn or soybean meal often have higher implied volatilities at higher strike prices.

Our research uses high-frequency data from the CME to investigate whether relaxing the assumption of a flat volatility surface changes considerably the price of LGM-Dairy insurance. While we find that relaxing the assumption of flat volatility influences price for insurance that would only offer protection against rise in corn feed costs (such as corn call options), when multiple commodity prices are insured at the same time (such is the case with LGM-Dairy), the assumption of flat implied volatility does not change premiums in a financially important way that would warrant amending the current LGM-Dairy pricing methodology.

Introduction

A constellation of recent policy changes, macroeconomic imbalances, structural shifts in production and an increasing reliance on export markets have resulted in increased volatility of profit margins in the U.S. dairy sector. A rich set of exchange-traded and over-the-counter financial products has been developed as a result of increased demand for risk management by dairy producers and industrial buyers of dairy products. More recently, increased volatility in the market prices for corn and other feed grains has resulted in a change in focus from milk price risk to income-over-feed-cost (IOFC) margin risk management. Given the capital intensity of milk production, the true risk consists not of month-to-month variations in IOFC margins, but from a possibility of prolonged periods of exceptionally low margins. As such, the optimal risk management instrument would be the one that protects margins, rather than individual input or output prices, and focuses on multi-month average margins, rather than margins in any particular month (Bozic, Newton, Thraen and Gould, forthcoming). Recognizing these aspects as salient risk dimensions, Hart, Babcock and Hayes (2001) proposed a method for pricing livestock revenue insurance using as an Asian basket type option. This method, with some modifications, is a foundation on which several margin insurance products were ultimately designed and offered to beef cattle, hog and dairy farm operators. A product designed specifically for dairy producers, labeled Livestock Gross Margin Insurance for Dairy Cattle (LGM-Dairy) was first offered in August 2008, sold through certified crop insurance agents, and underwritten by the U.S. Department of Agriculture's Risk Management Agency (RMA). LGM-Dairy has been continuously gaining popularity with dairy farmers since its introduction with the demand for this product currently being much greater than the amount of cwt. of milk that can be protected given the limited RMA budget for this product.

The official LGM-Dairy rating method stipulates that terminal prices for all commodities used to calculate insured IOFC margins are distributed lognormally, with mean equal to futures prices and variance determined by implied volatility obtained from at-the-money options, with both calculated at the time of contract purchase. The lognormality assumption may be most convenient, but has in fact been systematically refuted by many authors with respect to agricultural prices (e.g. Fackler & King, 1990; Kang & Brorsen, 1995; Sherrick, Garcia & Tirupattur, 1998). As is well known, lognormality of terminal prices is consistent with flat implied volatility curves, such that volatility implied from options prices does not change across strike prices. In contrast, if kurtosis of terminal price risk-neutral distribution is higher than would be the case under lognormality, implied volatility curve would be convex, the phenomenon known as volatility smile (Hull, 2009). Likewise, if terminal price risk-neutral distribution is more skewed than would be the case under lognormality, an upward volatility skew would emerge (Corrado & Su, 1997). Volatility curves for commodities consistently exhibit upward volatility skew. High skewness of commodity prices has been explained by the non-negativity constraint on commodity inventories (Deaton & Laroque, 1992; Geman, 2005; Pirrong, 2011, Bozic & Fortenbery, 2011). When stocks of a commodity are low, they can no longer effectively serve as a buffer against supply or demand shocks, rendering price more volatile. In other words, the volatility coefficient, rather than being constant, becomes a function increasing in commodity price. For that reason, Geman (2005) denoted this phenomenon as the *inverse leverage effect*. The importance of distributional assumptions for

pricing crop insurance has been recognized in the literature (e.g. Goodwin, Roberts &Coble, 2000; Sherrick et al., 2004). For example, Sherrick et al. (2004) found that alternative distributional assumptions regarding crop yields produce large differences in expected payouts from crop insurance products.

In this analysis we examine the what effect on LGM-Dairy premiums would there be if skewness and kurtosis of option-implied terminal price distributions were allowed to be determined by data, rather than restricted by a choice of option pricing model that stipulates lognormality. In other words, does accounting for volatility skews and smiles substantially influence actuarially fair premiums for LGM-Dairy insurance? Intuitively, presence of strong asymmetry or heavy tails may indicate higher likelihood of extreme events. Consequently, actuarially fair premiums for insurance products designed to protect only against catastrophic losses may be higher. As most users of LGM-Dairy treat this product as catastrophic margin risk insurance, it is important to understand the impact of distributional assumptions on LGM-Dairy premiums.

For the present analysis we utilize the Generalized Lambda distribution, a four parameter distribution that allows flexible modeling of the first four moments of price distribution. We use high frequency data for Class III milk, corn, and soybean meal to precisely estimate higher moments of prices implied from option premiums. Using Monte Carlo experiments we evaluate the impacts of excess asymmetry and heavy tails on actuarially fair insurance premiums for the LGM-Dairy margin insurance program. Anticipating the conclusion of our paper, we find that volatility skew effects do exist, but are fully eliminated by the basket option nature of the LGM-Dairy product. As such, we conclude that in calculation of the LGM-Dairy premiums excess asymmetry and heavy tails do not translate into bias of any financial importance. Financial instruments such as LGM-Dairy are complex and difficult to understand. This complexity can and does raise suspicion as to the fairness of the product by potential users. Our finding should serve to mitigate such concerns by both users and policy analysts.

In the following, we first provide an overview of the LGM-Dairy program, focusing on program elements important for the current analysis. The second section introduces Generalized Lambda distribution, and methods for estimating its parameters using high frequency futures and options data. In the third section we estimate the effect of flexible higher moments on LGM-Dairy premiums under a variety of insurance contract configurations. The final section contains several Monte Carlo experiments designed to explore the role of volatility smiles in detail.

An Overview of the Livestock Gross Margin Insurance Program for Dairy Cattle

LGM-Dairy is a revenue insurance product U.S. dairy farmers may use to protect income-over–feed-costs margin. Although the insurance product is privately owned, it is endorsed by the U.S. federal government, administered through USDA's Risk Management Agency, and covered liabilities are reinsured by the Federal Crop Insurance Corporation. Protected gross margin can be expressed as

$$G = \sum_{i=2}^{11} \left[\left(f_i^M - D \right) \times M_i - \left(f_i^C \times C_i + f_i^{SBM} \times SBM_j \right) \right] \quad (1)$$

where f_i^M, f_i^C, f_i^{SBM} denote respectively Class III milk, corn and soybean meal CME futures prices at time of contract purchase, and M_i, C_i, SBM_i are insured milk marketings, and corresponding corn and soybean meal feed amounts chosen by the policy buyer. Under LGM-Dairy up to 10 consecutive months, indexed by i in (1), can be insured on a single contract, excluding the first calendar month after the contract purchase. Deductible level D is allowed to vary between \$0.00 and \$2.00. The higher the deductible the more LGM-Dairy is used as a protection against large losses only. Maximum LGM-Dairy insurance coverage is limited to 24,000,000 lbs of milk per farm operation per 10 months or within a particular crop year, and as such, milk production expected from a dairy herd of up to approximately 1,200 milking cows can be fully covered.[1] LGM-Dairy insurance is designed to be sold once a month, on the last business Friday of the month, totaling 12 sales events per year. However, as insurance premiums as well as associated administrative and overhead fees are subsidized by the U.S. federal government, in 2011 and 2012 budget available for the program support has been insufficient to meet farmers' demand, limiting LGM-Dairy availability to 5 sales events in 2011 and only three in 2012 (through end of September 2012).

By rule, premiums for LGM-Dairy are based on expected indemnity, calculated through Monte Carlo simulations of 5,000 random price events (RMA, 2005). The premium is set at 1.03 times the average indemnity observed over the simulations. The 3% surcharge is administrative fee that supports a reserve fund at the Federal Crop Insurance Corporation. In order to simulate indemnities, a joint distribution function of terminal feed and milk prices over the forthcoming 10-month period must be defined. This joint distribution is built in two steps. First, futures and options-implied information are used to calculate moments of marginal price distributions, stipulated to be from the lognormal family.[2] In particular, expected terminal price is calculated as the three day average of relevant futures prices, immediately preceding the LGM-Dairy sales event. Terminal log-price variance is calculated as the square of the three-day average implied volatility of at-the-money call and put options multiplied by the time left to realized price determination.

While Class III milk futures trade for each calendar month, corn only has 5 contracts per year, and soybean meal has 8. In total, up to 24 marginal distributions are fitted directly based on futures and options data. The procedure developed by Iman and Conover (1982) is used to couple marginal distributions into joint distribution function using Spearman's rank correlation coefficients of historical futures price deviates. Price deviates over 1978-2005 are used to calculate Spearman's correlation coefficients for corn and soybean meal. To calculate correlations among milk prices, the time period 1998-2005 is used. To overcome the non-positive definiteness in the full correlation matrix, correlations are assumed to be zero between milk and corn and between milk and soybean meal prices. After simulating 24 correlated

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[1] *The unit of analysis when measuring milk production is the number of 100 lbs (hundred weight, cwt) of milk produced. Thus the 24,000,000 lbs represent 24,000 cwt.*

[2] *To account for USDA's Dairy Products Price Support program simulated Class III milk price is never allowed to fall below \$8.50. For more detail refer to RMA (2005).*

marginal distributions, simulated prices for months for which corn or soybean meal futures contracts do not trade are calculated as weighted average of simulated prices for surrounding months. More detailed description of LGM-Dairy rating method can be found in Gould and Cabrera (2011) and RMA (2005).

Similar to basket options, LGM-Dairy insurance compensates producers only for adverse relative movements of milk and feed prices, resulting in much cheaper premiums compared to the use of options to establish the IOFC floor. Similar to Asian options that settle against an average price of the underlying asset, LGM-dairy allows insuring margins averaged over up to 10 consecutive months, with considerable savings compared to a sequence of basket options or option bundles.

In order to examine the impact of volatility smiles on LGM-Dairy premiums, we need to be able to estimate flexible distributional forms for terminal prices. Furthermore, in Monte Carlo simulations, it will help us to be able to control for mean and variance, and increase only skewness or kurtosis of terminal price. For these reasons, we need a distribution with at least four parameters, to allow enough degrees of freedom to control each moment without altering others. One such distribution is Generalized Lambda distribution (GLD) introduced by Ramberg and Schmeiser (1974) and applied to option pricing by Corrado (2001). As this distribution is not often used, we present several GLD features in some depth.

Introduction to the Generalized Lambda Distribution

In 1960, John Tukey introduced a one-parameter lambda distribution (Tukey 1960). The primary advantage of Tukey's lambda distribution, in times of scarce computational power, was its closed-form quantile function, i.e. the inverse of the cumulative distribution function. It was generalized by Ramberg and Schmeiser (1972) to a three-parameter distribution with location, scale and shape parameters. This was followed by Ramberg and Schmeiser (1974) who added an additional shape parameter so as to facilitate Monte Carlo generation of asymmetric random variables. Clear expression for the first four moments of the distribution was given in Ramberg et. al. (1979). In a book dedicated to extensive treatment of GLD, Karian and Dudewicz (2000) explain how GLD can be used to approximate many common distributions such as normal, lognormal, Gamma, Weibull, Beta, logistic, Pareto, etc.

The GLD family is defined by its quantile function (i.e. inverse cumulative distribution function) which can be represented via the following:

$$Q(y) = Q(y;\lambda_1,\lambda_2,\lambda_3,\lambda_4) = \lambda_1 + \frac{y^{\lambda_3} + (1-y)^{\lambda_4}}{\lambda_2} \tag{2}$$

where $0 \le y \le 1$. Given (2) the probability density function at $x = Q(y)$ is

$$f(x) = \frac{\lambda_2}{\lambda_3 y^{\lambda_3 - 1} + \lambda_4 (1-y)^{\lambda_4 - 1}} \tag{3}$$

Under restrictions that $\lambda_3 > -\frac{1}{4}; \lambda_4 > -\frac{1}{4}$, the first four moments $(\alpha_i,\ i = 1,\ldots,4)$ are well defined and are given by

$$\alpha_1 = \mu = E\left(X\right) = \lambda_1 + \frac{A}{\lambda_2} \tag{4}$$

$$\alpha_2 = \sigma^2 = E\left[\left(X-\mu\right)^2\right] = \frac{B - A^2}{\lambda_2^2} \tag{5}$$

$$\alpha_3 = S = \frac{E\left(X - E(X)\right)^3}{\sigma^3} = \frac{C - 3AB + 2A^3}{\lambda_2^3\sigma^3} \tag{6}$$

$$\alpha_4 = K = \frac{E\left(X - E\left(X\right)\right)^4}{\sigma^4} = \frac{D - 4AC + 6A^2B - 3A^4}{\lambda_2^4\sigma^4} \tag{7}$$

where S represents skewness, K represents kurtosis and

$$A = \frac{1}{1+\lambda_3} - \frac{1}{1+\lambda_4} \tag{8}$$

$$B = \frac{1}{1+2\lambda_3} + \frac{1}{1+2\lambda_4} - 2\beta\left(1+\lambda_3, 1+\lambda_4\right) \tag{9}$$

$$C = \frac{1}{1+3\lambda_3} - \frac{1}{1+3\lambda_4} - 3\beta\left(1+2\lambda_3, 1+\lambda_4\right) + 3\beta\left(1+\lambda_3, 1+2\lambda_4\right) \tag{10}$$

$$D = \frac{1}{1+4\lambda_3} + \frac{1}{1+4\lambda_4} - 4\beta\left(1+3\lambda_3, 1+\lambda_4\right) + 6\beta\left(1+2\lambda_3, 1+2\lambda_4\right) - 4\beta\left(1+\lambda_3, 1+3\lambda_4\right) \tag{11}$$

and $\beta\left(a,b\right)$ is the complete beta function defined by

$$\beta\left(a,b\right) = \int_0^1 x^{a-1}\left(1-x\right)^{b-1}\,dx. \tag{12}$$

In generating random variables with desired moments μ, σ^2, S, K we first numerically calculate parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ then exploit the closed form quantile function (2) for simulation of random draws. Parameter λ_1 determines location, λ_2 determines scale, and shape, i.e. skewness and kurtosis are determined solely by λ_3 and λ_4. Having four parameters we can adjust each of the first four moments without affecting the other three, as long as the resulting change does not fall in the area not covered by GLD. Corrado (2001) developed a formula for pricing European options on stocks using GLD. Bozic and Fortenbery (2011) extended that approach to pricing options on futures.

Given futures price f_t, strike K, risk-free interest rate r, time to maturity $\tau = \frac{T-t}{365}$, implied volatility σ and shape parameters λ_3 and λ_4 and denoting the GLD cumulative density function with $F(x)$, the value of European call option is

$$C = f_t e^{-r\tau} G_1 - e^{-r\tau} K G_2 \tag{13}$$

where $G_2 = 1 - F(K)$ and

$$G_1 = f_t \left(1 - F(K) + \frac{\sqrt{e^{\sigma^2\tau} - 1}}{sign(\lambda_3)\sqrt{B - A^2}} \left(\frac{F(K) - F(K)^{\lambda_3+1}}{\lambda_3 + 1} + \frac{1 - F(K) - (1 - F(K))^{\lambda_4+1}}{\lambda_4 + 1} \right) \right) \tag{14}$$

Although the concept of implied volatility only has a direct connection to terminal price variance in the context of geometric Brownian motion that underpins Black-Scholes option pricing model, Bozic and Fortenbery (2011) include σ in (14) to make GLD option pricing model an approximate generalization of Black's model, such that $\sigma^2\tau$ is indeed the variance of terminal log-prices in case that higher moments conform to those implied by the lognormal distribution.

Estimating Parameters of Generalized Lambda Distributions

We are interested in calculating LGM-Dairy premiums with marginal distributions for milk, corn and soybean meal being from the GLD family, with moments estimated based on traded options data. Estimating higher moments of option-implied distributions using end-of-day settle prices may bias estimates of moments in two ways. First, options price is assumed to be based on the latest observed futures price, but with end of day data we have no way of knowing what was the last time an option traded, and what was the futures price at that moment. Secondly, settlement procedures may induce further bias. For example, for Class III milk options, CME regulations stipulate that dairy options are settled using flat volatility surface determined by the at-the-money straddle (CME, 2012). In order to avoid such biases implied skewness and kurtosis using the GLD option pricing model are estimated using high frequency futures and options data. Times and sales data were obtained for Class III milk, corn and soybean meal futures and options contracts for 2011. To avoid market microstructure noise we restricted data frequency to be no less than 15 minute interval for each strike. Rather than using a single value for futures price in a day, we matched each options contract transaction with last observed futures contract transaction that immediately preceded it. Three days of matched options-futures data were pooled and moments of GLD were then estimated using nonlinear least squares, minimizing the sum of squared option pricing errors.

Although LGM-Dairy did not trade in all months of 2011 due to program budget constraints, we perform our analysis as if LGM-Dairy was offered in all calendar months. With four GLD parameters estimated per terminal price distribution, 22 to 24 marginal distributions per sales event, and assumed 12 LGM-Dairy sales events in 2011, our analysis required the estimation of 1,080 lambda parameters. As such it is not feasible to present all estimation

results in this paper. Instead, we illustrate our approach by presenting data used for LGM-Dairy premium determination for the purchase of the May 2011 contract offering in Table 1. In this table we show the marginal distributions moments assuming lognormality as well as moments implied from GLD option pricing model. Consistent with previous research, we find that skewness estimates for both corn and soybean prices generally exceed lognormality-consistent skewness coefficients. Surprisingly, kurtosis estimates for distant-delivery months were found to exhibit lower kurtosis than consistent with lognormality. This may be artifact of small data sample, i.e. low number of transactions used in estimation of implied distributions close to one year to maturity.

Volatility Smiles and LGM-Dairy Premiums

A complete definition of LGM insurance policy requires a choice of insured milk marketings in each of 10 insurable months, as well as declared feed amounts and margin deductible. For our analysis, we build an insurance policy profile for a dairy farm with 500 milk cows, producing 9,000 cwt. of milk per month. National Milk Producers Federation (2010) proposed a feed ration that accounts for all feeding needs of a dairy herd, including milking cows, dry cows, hospital cows and replacement heifers. With some modifications, this ration is likely to be adopted by U.S. Congress as an appropriate input to calculation of IOFC margin in 2012 Farm Bill legislation . The assumed ration consists of 1.0728 bushels of corn, 0.00735 tons of soybean meal and 0.0137 tons of alfalfa hay per cwt of milk produced. Unlike corn and soybean meal, alfalfa hay does not trade on organized futures markets. In order to obtain a measure of expected future alfalfa hay costs, we regressed monthly alfalfa hay price received by U.S. farmers on monthly prices received for corn and soybean meal, with all three prices expressed as dollars per ton. The estimation period used is January 2005-June 2012. Regression results are given in Table 2. Based on regression results, a ton of alfalfa hay is converted to 0.727 tons of corn and -0.135 tons of soybean meal. Given the proposed utilization of alfalfa hay listed above, after conversion has been performed, the final corn and soybean meal equivalents per hundredweight of milk are 0.0401 tons of corn and 0.005503 tons of soybean meal per hundredweight of milk produced. We add $1.70 milk price basis to compensate for the difference between farm-level milk prices (i.e., the "All milk" price) and the Class III milk futures contracts. The intercept from the regression of hay on corn and soybean meal prices, multiplied by the utilization of alfalfa hay per hundredweight of milk subtracts $0.995 from the basis. Given these specifications, time-*t* expected per cwt. income over feed cost margin $E_t(\text{IOFC}_i)$ for insurable month i is given by

$$E_t\left(\text{IOFC}_i\right) = \$0.705 + f_{t,i}^{M} - 0.040141 f_{t,i}^{C} - 0.005503 f_{t,i}^{SBM} \tag{14}$$

where $f_{t,i}^{M}$, $f_{t,i}^{C}$, $f_{t,i}^{SBM}$ are futures prices for Class III milk, corn and soybean meal, as observed at time *t* and expressed in dollars per ton, for contracts expiring *i* months later. For this analysis we assume the LGM-Dairy insurance policy is bought every month, and that 1/3 of expected milk marketings are insured for the 4th, 5th and 6th insurable months. This strategy, if implemented continuously, protects 100% of expected milk marketings.

Although the maximum allowable deductible is $2.00, in order to examine the impact of asymmetry and heavy tails, we evaluate premium costs under levels of deductible chosen that vary from $0.00 to $5.00. Protected margins at $5.00 deductible correspond to levels observed in the midst of 2009 slump in milk prices that is considered to have been a very rare event. As such, we are able to identify the effect of skewness and kurtosis on premiums for a set of policies that vary from those that cover shallow losses to policies that provide protection only in truly catastrophic scenarios.

Table 3 is used to provide results of our Monte Carlo experiments. In each scenario illustrated, we calculated LGM-Dairy premiums averaged over 12 simulated LGM-Dairy sales events in 2011. Columns (2) and (3) are used to show the gross margin guarantee, both total and per hundredweight of milk produced under alternative deductible levels. It should be noted that the gross margin guarantee at $0.00 deductible is equal to expected gross margins at sign-up. Average premiums obtained using the official RMA rating methods are presented in columns (4) and (5). In order to evaluate whether or not the GLD can reliably approximate the situation where marginal distributions are indeed lognormal, in column (6) we list average premiums under GLD marginal distributions, with GLD parameters chosen in such way that mean, variance, skewness and kurtosis are matched with moments from the official RMA method with lognormal marginal distributions.[3]

GLD replicates tend to have slightly lower premiums, but with premium differences less than half of one percent of original premiums. LGM-Dairy premiums obtained by using marginal distributions that rely on implied moments obtained through estimated GLD parameters using high-frequency data are given in column (7). We observe only minor changes compared to premiums calculated under the original RMA method.

Finally, in column (8) we evaluated the consequences on premiums would be if expected price distributions for corn and soybean meal had skewness that is 70% higher (more positive) than skewness consistent with lognormality. Kurtosis is likewise increased by 60%. With strong asymmetry and heavy tails, likelihood of extremely high feed prices is increased, and as a consequence premiums for policies that protect against extremely low margins should also increase. We indeed find in column (8) that premiums for very high levels of deductible are higher than the official RMA-method based premiums. In contrast, allowing for asymmetry and leptokurtosis actually reduce premiums for low-deductible policies. In order to increase density in far right tail of feed price distributions, without changing mean or variance, density must decrease for prices just above the mean. Likewise, an increase in asymmetry, while fattening the right tail of the price distribution, increases density just below the mean as well. Rather than producing an across-the-board increase in premiums, asymmetry and heavy tails reduce premiums for shallow-loss policies, and increase premiums for catastrophic-loss-only coverage. Even more striking is the magnitude of change in premiums. Despite substantial increase in higher moments, premiums never change by more than 2.6%. This is surprisingly miniscule

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[3] *Lognormal distribution is fully defined by parameters for mean and variance, and skewness and kurtosis can be directly calculated based on these parameters. We calculated skewness and kurtosis of lognormal distributions then found GLD parameters that produce the same higher moments as under lognormality.*

impact, compared to effects of alternative yield distributions on crop insurance premiums, as reported by Sherrick et al. (2004).

To explore this issue further, we built an artificial scenario wherein milk and soymeal prices are known with certainty and only source of risk arises from corn prices. Furthermore, we assume that a single month of IOFC margin is insured. We assume December 2011 IOFC margin is insured during the May 2011 LGM sales event. We designed four scenarios: 1) Corn price skewness increased by 50%; 2) Corn price kurtosis increased by 50% and 3) Both corn price skewness and kurtosis increased by 50%. While in the first three scenarios milk price is considered as deterministic, in the final scenario we let milk price be lognormally distributed, as in the original RMA method.

The results are presented in Figure 1. From scenario 1, we learn that increase in skewness, when corn price is the only source if IOFC margin risk, does increase premium costs substantially, and that premium increase is higher when LGM-Dairy is used to protect against catastrophic risks only. Isolated increase in kurtosis, as in scenario 2, is found to reduce premiums for deductibles lower than $4.40, and increase premiums thereupon. Reduction is the highest for policies with $2.00 deductible. Combined increase in skewness and kurtosis, examined in Scenario 3, behaves as a combination of Scenarios 1 and 2, with premiums reduced for low deductibles, and increased for high deductibles. While magnitude of premium changes in scenarios with only one source of risk is substantial, adding milk price as a second source of risk, uncorrelated with corn price per official RMA rating method, was sufficient to mostly neutralize the effect of increased skewness and/or kurtosis.

While in Scenario 1 we observe premium increases higher than 20% for high enough deductibles, adding milk price as a second source of risk made premiums more robust with respect to higher moments, with changes never exceeding 3%. We conclude that basket option nature of LGM-Dairy suffices to make the lognormality assumption a useful heuristic that does not bias LGM-Dairy premiums in any financially important way. In the full RMA Monte Carlo experiment, it seems that excess skewness and kurtosis only changed higher moments of indemnities. For example, over 5000 simulated scenarios, the maximum simulated indemnity for May 2011 sales event $147,176 under official RMA method, and $183,044 under the *Extreme Skewness and Kurtosis* scenario. However, average indemnity over 5000 simulated scenarios barely changed, and consequently LGM-Dairy premiums remained stable.

Conclusions

The recently introduced Livestock Gross Margin Insurance for Dairy Cattle (LGM-Dairy) is a highly popular insurance tool that enables U.S. dairy producers to protect against prolonged periods of substantial declines in dairy-related income over feed cost- margins. However, after four years of pilot-program status, LGM-Dairy has generated premium revenue that exceeds indemnity payments by more than thirty to one. That discrepancy has raised questions as to the actuarial fairness of LGM-Dairy rating method (Novakovic, 2012).

One potential source of LGM-Dairy mispricing may be assumptions regarding marginal price distributions. LGM-Dairy premiums are based on information contained in futures prices and options premiums, which are used to fit moments of marginal milk and feeds price distributions, assumed to be lognormal. A substantial body of evidence exists that refutes constant volatility models for futures prices and theory of commodity price dynamics suggests that commodity storability produces upward-bending skews in implied volatility curves. These volatility skews can be understood as indication that skewness of terminal price distribution exceeds skewness that would be consistent with lognormality.

In this article we used high-frequency data for Class III milk, corn and soybean meal futures and options to document the extent the existence and magnitude of volatility smiles and skews in milk and feed prices in 2011. Using Monte Carlo experiments we examined the effect of accounting for extreme excess skewness and kurtosis on LGM-Dairy premiums. We found no effect of any significant financial importance. Further experiments revealed that basket option nature of LGM-Dairy suffices to neutralize the premium-enhancing effect of excess skewness.

As this research has demonstrated, volatility skews and smiles in corn and soybean meal implied volatilities do not seem to change LGM-Dairy premiums sufficiently to warrant amending the LGM-Dairy rating method.

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Table 1. Comparison of LGM-Dairy Marginal Price Distributions for May 2012 Sales Event

	Official LGM-Dairy Rating Method (Lognormality)				Alternative LGM-Rating Method (Generalized Lambda)			
Commodity/ Month	Expected Price	Variance	Skewness	Kurtosis	Expected Price	Variance	Skewness	Kurtosis
Milk								
Jul	19.06	1.52	0.23	3.01	19.14	1.57	0.16	2.95
Aug	18.96	2.08	0.33	3.07	18.99	2.09	0.16	3.24
Sep	18.76	2.25	0.35	3.11	18.72	2.43	0.01	3.20
Oct	18.21	2.74	0.44	3.28	18.12	2.83	0.28	4.03
Nov	17.85	2.87	0.50	3.40	17.74	3.14	0.77	4.43
Dec	17.31	2.96	0.54	3.35	17.23	2.89	0.42	3.28
Jan	16.92	3.05	0.57	3.38	16.87	3.11	0.53	3.16
Feb	16.54	3.15	0.60	3.43	16.50	3.12	0.52	3.10
Mar	16.44	3.48	0.68	3.63	16.31	3.43	0.64	3.30
Apr	16.40	3.61	0.68	3.62	16.35	3.64	0.61	3.18
Corn								
Jul	7.49	1.06	0.43	3.32	7.48	0.83	0.38	3.27
Sep	7.19	1.57	0.62	3.54	7.16	1.46	0.70	3.93
Dec	6.79	1.84	0.80	4.17	6.71	1.83	0.98	4.28
Mar	6.89	2.21	1.02	5.03	6.77	2.26	1.20	5.24
May	6.98	2.32	0.94	5.10	6.93	2.35	1.04	3.64
Soybean Meal								
Jul	358.46	28.92	0.23	3.13	358.81	21.71	0.44	3.00
Aug	358.49	39.84	0.39	3.16	359.35	35.89	0.91	3.93
Sep	358.06	50.02	0.43	3.30	356.98	46.69	0.96	3.87
Oct	352.86	52.96	0.44	3.38	350.47	51.66	1.11	3.81
Dec	354.22	60.24	0.49	3.40	351.24	57.84	0.97	4.05
Jan	355.29	64.57	0.57	3.55	354.61	65.04	0.53	3.10
Mar	356.10	71.58	0.65	3.79	353.61	72.10	0.65	3.41
May	352.29	78.08	0.72	4.03	353.77	80.02	0.63	3.07

Table 2. Regression of U.S.Alfalfa Hay Prices ($/ton) on U.S.Corn and Soybean Meal Prices

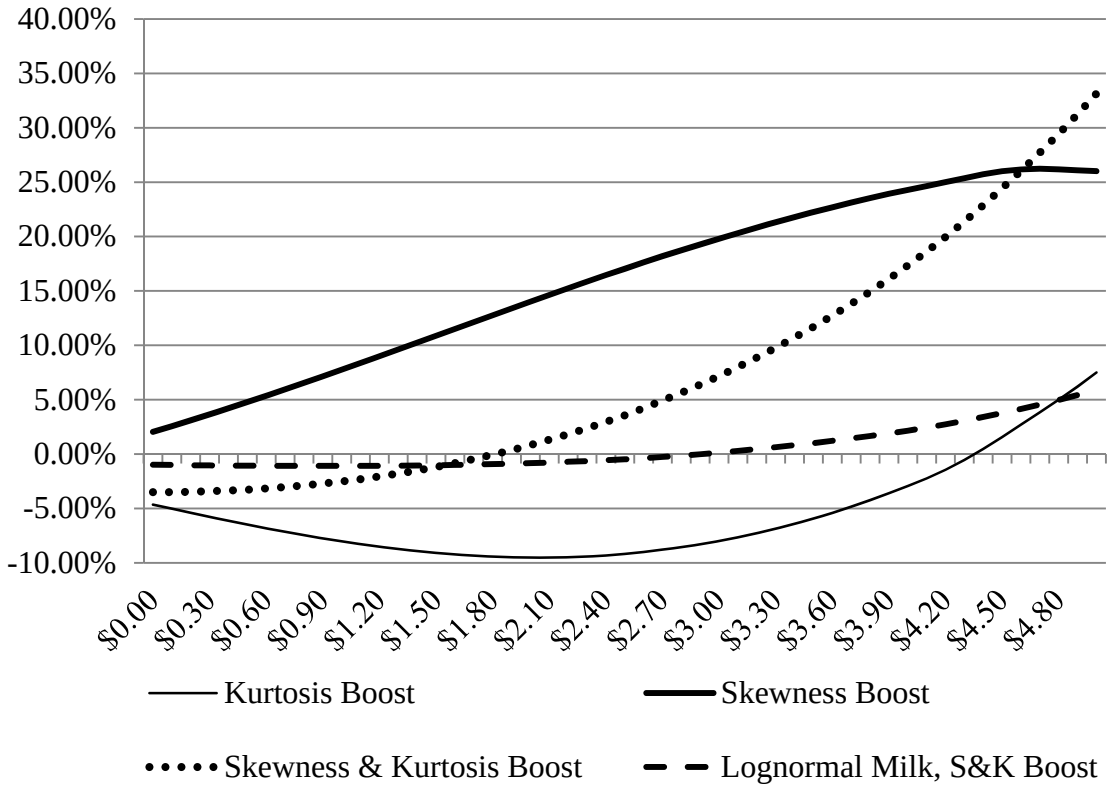
Variable	Coefficient	Standard Error	T-statistic
Intercept	72.639	5.703	12.736
Soybean meal price ($/ ton)	-0.135	0.031	-4.300
Corn price ($/ton)	0.727	0.047	15.551
Number of Observations	90		
R^2	0.82		

Table 3. LGM-Dairy Premiums under Alternative Marginal Distribution Assumptions

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
			Distribution Assumption							
Deduc-tible	Gross Margin Guarantee		RMA Official Rating Method		RMA Official Rating Method Under GLD		GLD Marginal Distributions		Extreme Feed Skewness and Kurtosis	
($/cwt)	($)	($/cwt)	($)	($/cwt)	($)	% Change	($)	% Change	($)	% Change
$0.00	55,779	6.20	13,783	1.53	13,737	-0.33	13,668	-0.83	13,538	-1.78
$0.25	53,529	5.95	12,650	1.41	12,602	-0.38	12,544	-0.84	12,416	-1.85
$0.50	51,279	5.70	11,585	1.29	11,536	-0.42	11,487	-0.85	11,364	-1.91
$0.75	49,029	5.45	10,589	1.18	10,540	-0.46	10,501	-0.83	10,385	-1.93
$1.00	46,779	5.20	9,659	1.07	9,612	-0.49	9,580	-0.82	9,475	-1.90
$1.25	44,529	4.95	8,793	0.98	8,748	-0.51	8,724	-0.78	8,631	-1.84
$1.50	42,279	4.70	7,989	0.89	7,948	-0.51	7,930	-0.74	7,849	-1.75
$1.75	40,029	4.45	7,244	0.80	7,206	-0.52	7,195	-0.68	7,127	-1.62
$2.00	37,779	4.20	6,554	0.73	6,520	-0.52	6,515	-0.60	6,461	-1.42
$2.25	35,529	3.95	5,916	0.66	5,887	-0.49	5,886	-0.51	5,849	-1.13
$2.50	33,279	3.70	5,328	0.59	5,303	-0.47	5,307	-0.39	5,287	-0.77
$2.75	31,029	3.45	4,789	0.53	4,768	-0.44	4,776	-0.27	4,772	-0.35
$3.00	28,779	3.20	4,293	0.48	4,277	-0.37	4,288	-0.12	4,300	0.16
$3.25	26,529	2.95	3,839	0.43	3,827	-0.31	3,842	0.08	3,868	0.76
$3.50	24,279	2.70	3,423	0.38	3,416	-0.20	3,434	0.32	3,476	1.55
$3.75	22,029	2.45	3,046	0.34	3,045	-0.03	3,064	0.59	3,121	2.46
$4.00	19,779	2.20	2,705	0.30	2,707	0.07	2,728	0.85	2,798	3.44

Note: Analysis for a dairy farm with 500 cows, marketing 9,000 cwt of milk each month. LGM-Dairy insurance policy is purchased every month, with one third of the marketings insured for 4th, 5th and 6th insurable month. Declared feed amounts chosen to approximate income over feed margin proposed in 2012 Farm Bill. Extreme feed skewness and kurtosis scenario has marginal distributions with same means and variances as in RMA official method, but with skewness increased by 60% and kurtosis by 70%.

Figure 1. Effects of Increase in Corn Price Skewness and Kurtosis on LGM-Dairy Premiums



Note: We built an environment where milk and soybean meal prices are assumed to be deterministic, and only source of risk arises from uncertainty regarding corn prices. Three scenarios (Kurtosis Boost; Skewness Boost; and Skewness & Kurtosis Boost) examine the impact of increase in higher moments of corn prices, while the last scenario (Lognormal Milk, S&K Boost) allows additional uncertainty from milk prices.