Program on Dairy Markets and Policy Working Paper Series

Price Discovery, Volatility Spillovers and Adequacy of Speculation in the U.S. Dairy Markets

Working Paper Number 14-01

February 2014

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ABSTRACT

This article contributes to the debate on time series properties of commodity cash and futures markets and the impact of speculation on commodity futures markets. We reconcile production theory, which predicts cash commodity prices will be mean-reverting, with the efficient market hypothesis which is consistent with unit root process for futures prices. It is shown that when the underlying cash price series does not contain a unit root, a nearby futures price series can be nonlinear, having martingale properties within each contract segment, and mean-reverting changes at contract rollover points. We develop a novel ECM-BEKK-MEX model that handles nonlinearities in futures prices in a simple and practical way, and allows full flexibility in modeling the impact of speculation on conditional cash and futures price variances while preserving positive definiteness of the bivariate variance matrix. The model is applied to the U.S. dairy sector. Results suggest cheddar cheese futures markets represent the primary center for cheese price discovery, although both information flows and volatility spillovers are bidirectional. Net long speculative positions do not forecast the direction of change of futures prices, but more speculation, as measured by Working's T index, is associated with a lower conditional variance of futures prices in the next period.

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Introduction

In the aftermath of the major commodity price shocks of 2007 and 2008 there has been considerable interest in the performance of commodity futures markets. Masters (2008) has argued that speculative activity was "*one of, if not the primary factor*" affecting commodity prices. The sentiment that excessive speculation should be curbed even led to recent policy changes. The U.S. Commodity Futures Trading Commission (CFTC) voted in 2011 to impose lower limits on the number of contracts speculators can hold in 28 commodity futures markets.

The hypothesis that speculators, rather than fundamental market factors, were behind dramatic commodity price shocks was subsequently challenged by many authors (e.g. Hamilton 2009; Irwin, Sanders and Merrin 2009; Brunetti, Buyuksahin and Harris 2011; Irwin and Sanders 2012). However, most of the studies on this topic have focused on institutional investors that passively hold long-only positions, commonly referred to as Index Funds. In addition, they focus on futures markets with relatively large volume.

In thin futures markets the volume may not be sufficiently high to generate trade interest of large institutional investors. In these markets most speculation is likely coming from smaller, non-professional speculators. Consequently, speculation may represent a smaller total market share relative to deeper markets (Fortenbery 2011). However, the fact that the speculators in thin markets may represent a less informed trader that relies more on technical versus fundamental market information to generate trade activity implies their trade decisions may exacerbate the level of market noise (Fortenbery and Zapata, 2004). When noise traders are present, even more informed speculators may contribute to higher market volatility as their trade decisions take into account not just market fundamentals, but the anticipated reaction of noise traders to past price movements (de Long et al. 1990).

Dairy futures markets satisfy the conditions indicated by the authors cited above that could lead to destabilizing speculation. Dairy futures markets are relatively thin. Less than 10 percent of total U.S. milk production is hedged, and total open interest, while growing over time, is still very small.[1] In addition, dairy markets have a long history of determining cash milk prices milk off very thin spot markets for derived products (cheese, butter and nonfat dry milk) (GAO 2007). In such an environment it is conceivable that speculation-induced volatility in futures may lead to reduced cash price stability even when fundamental factors do not warrant price changes.

[1] For example, on September 24, 2013 Class III milk futures and options (the most liquid of the dairy markets) reported open interest of only 36,821 contracts. In contrast, the corn market reported a total open interest of just under 1.7 million contracts. Further, professionally managed money and swap dealers represented almost 40 percent of total long corn open interest, and over 31 percent of short open interest. Small non-reportable traders (which include small speculators and hedgers) accounted for about 10 percent of long open interest and 17 percent of the short side. Commercial firms (primarily hedgers) represented 23 percent of the long and 26 percent of the short corn open interest. In the Class III market, professionally managed money and swap dealers accounted for only 3.8 percent of long open interest, and 12.7 percent of the short side. Non-reportable traders accounted for 11 percent of long open interest and 25 percent of short interest. Commercial firms dominated accounting for over 51 percent of the long side 37 percent of short open interest.

This paper makes three contributions to the literature. The first contribution regards the the stationarity properties of futures and cash prices. Analyses of the impacts of futures speculation on price performance in cash and futures markets requires modelling of the joint cash and futures price and volatility dynamics. Correct model specification of joint price dynamics requires careful consideration of stationarity assumptions. Since Samuelson (1965) it has been understood that unbiased futures markets will behave as martingale process, i.e. a time series of futures prices will contain a unit root. For this reason, applied works where cash-futures relationships are analyzed tend to rely on the assumption that both futures and cash prices behave similarly. In other words, cash prices are also treated as unit root processes. This assumption allows the utilization of standard nonstationary time series methods (e.g. Bessler and Covey 1991; Schroeder and Goodwin 1991; McKenzie and Holt 2002; Frank and Garcia 2009).

However, commodity price theory does not predict permanent shocks to equilibrium cash prices, nor does it require cash prices to behave as a non-stationary process (Williams and Wright 1991; Deaton and Laroque 1992; Wang and Tomek 2007; Pirrong 2012). In this paper conditions are presented under which cash commodity prices behave as a stationary series, and the implications on measuring price dynamics between cash and continuous nearby futures prices are presented. We show that when cash prices do not have a unit root, nearby futures prices can be nonlinear, having martingale properties within each contract segment, and mean-reverting changes at contract rollover points. We then build an error-correction model able to handle these conditions in a simple and practical way.

Our second contribution is a new model for analyzing impacts of speculative activities on the conditional variances of cash and futures prices. Where GARCH-type models can be used to examine volatility dynamics, a BEKK specification is often employed to guarantee a positive definite covariance matrix (Engle and Kroner, 1995). While BEKK-type approach can accommodate additional regressors in the variance model (e.g. a measure of speculative activity), the quadratic form-based approach does not allow marginal impacts of added regressors on the conditional variance to be negative. This presents a major problem when modeling the impact of speculators on conditional covariance matrix of cash and futures prices, as it does not allow the model to reveal a potentially stabilizing effect of speculation. We develop a novel augmented BEKK-type model (referred hereafter as BEKK-MEX) that allows full flexibility in modeling the determinants of conditional cash and futures price variance while preserving positive definiteness of the bivariate variance matrix.

The final contribution is the application of an error-correction model with BEKK-MEX volatility dynamics to analyze the information flows and volatility spillovers between cash and futures dairy markets, as well as the effects of futures speculation on the stability of cash and futures prices for milk.

The paper proceeds as follows. The first section expands on the discussion of the properties of cash and futures prices. Section two presents unit root tests of the dairy prices used in the empirical analysis. In the third section, the ECM-BEKK-MEX model is introduced. Section four contains the results of the econometric analysis, followed by a discussion of findings and suggestions for further research.

Price dynamics in spot and futures commodity markets

In order to appropriately model the information flow between cash and futures markets, it is important to understand the time series properties of both cash and futures prices. In particular, when prices are non-stationary, estimating models in price levels may result in spurious regressions (Granger and Newbold 1974; Phillips 1986). In addition, estimating models with differenced series might also result in a misspecified model, and it would be more appropriate to utilize the cointegration framework (Hamilton, 1994).

Theoretical priors regarding time series properties of agricultural commodity cash and futures prices are remarkably different. Structural models of commodity price dynamics suggest non-linear cash price processes characterized by occasional price spikes that are eventually dissipated unless markets have gone through a structural change (Williams and Wright 1991; Deaton and Laroque 1992; Pirrong 2011). Tomek and Wang (2007) review the literature on unit roots in commodity cash prices and find that unit root test results are sensitive to the specification of the test question. More recent research reveals that once structural breaks and nonlinearities are taken into account, tests often reject unit root hypothesis of cash commodity prices (Balagtas and Holt 2009; Enders and Holt 2012). In addition, analyses of the term structure of futures prices suggests cash prices are perceived by market participants as mean-reverting (Bessembinder et al 1995, Bozic et al, 2012).

Rather than debating the cash price properties on statistical grounds, it will be more productive to summarize the implications from economic theory. The fundamental property of cash prices emerging in perfectly competitive markets is the necessity of zero long-run economic profits for the marginal producer. This implies profit margins over time behave as mean-reverting time series. If the long-run industry average cost curve is flat (a case of constant returns to scale), any shift in the demand function will produce only a temporary shock to cash prices. However, even with constant long run average costs, if production cannot adjust quickly to demand shocks in the short run, cash prices may exhibit high a degree of persistency and rather slow reversion to long-run averages. For example, with annually harvested commodities, demand shocks cannot be compensated for by increases in supply prior to the next harvest (assuming no imports from outside the market region). If returns to scale are decreasing, then shifts in demand will manifest as permanent shocks to cash prices. Finally, permanent changes in input costs will shift the long-run average cost curve and thus induce a structural change in cash price series.

While the time series properties of cash prices are argued based on production theory, the time series characteristics of futures prices emerge from the theory of finance (Samuelson, 1965). If a futures market is efficient (i.e. if futures prices fully account for all available information) then prices within a single contract will be martingales if the marginal risk premium is zero, submartingales if marginal risk premium is positive (i.e. futures are downward biased and traders having long positions are rewarded), and supermartingales if the marginal risk premium is negative (i.e. futures are upward biased and traders having short positions are rewarded). In any case, by deducting the marginal risk premium we can arrive at a martingale series whose direction of change cannot be predicted based on currently available information. From this it follows that whether the risk premium is present or not, efficient futures prices will be nonstationary, i.e. shocks to futures prices will to be permanent.

While the efficient market hypothesis has clear implications for time series behavior of futures prices within a single contract, the collection of observed futures prices for differing delivery dates constitutes a panel of partially overlapping time series (Smith, 2005). This data structure does not permit the use of standard unit root tests, so a common practice is to form a single futures price time series by choosing only one price observation per unit of time. A continuous futures price series is constructed by choosing segments from consecutive contracts at the time when each contract was the *j*th contract to maturity, as illustrated by Figure 1 for the 1st nearby series. It has been recognized that conflating the panel of futures prices to a single price series may induce complicated nonlinear dynamics in the data (Smith, 2005).
For example, consider a situation where the underlying cash prices are mean-reverting. Suppose a cash price series for a commodity is second-order stationary. Let μ be the unconditional mean of the cash price, and σ_c^2 the unconditional variance. By the Wold decomposition theorem (Wold, 1954), there exists the unique fundamental moving average representation of the cash price stochastic process:

$$c_t = \mu + \sum_{i=0}^{\infty} \alpha_i \varepsilon_{t-i} \tag{1}$$

where $\alpha_0 = 1, E(\varepsilon_t) = 0, \mathrm{E}(\varepsilon_t \varepsilon_{t-k}) = 0, \forall\, \mathrm{k}$. Suppose that a futures contract is written for that commodity, and for simplicity, assume that there is no basis at futures contract expiry, i.e. the terminal futures price equals the cash price at the contract expiration. Finally, assume that futures prices are efficient and embody no risk premium. Denote futures price at time t for a contract that expires at time T_k by $f_t^{T_k}$. Let the first nearby futures price series be constructed by rolling contracts over one day before the expiration date:

$$\left\{ f_{j=1} \right\} = \left(f_1^{T_1}, \ldots, f_{T_1-1}^{T_1}, f_{T_1}^{T_2}, \ldots f_{T_2-1}^{T_2}, f_{T_2}^{T_3}, \ldots \right) \tag{2}$$

Under these assumptions, it can be shown that the following facts hold:

$$E_t\left[f_{t+1}^{T_k} \right] = f_t^{T_k} \tag{3}$$

$$E_{T_k-1}\left[f_{T_k}^{T_{k+1}} \right] = f_{T_k-1}^{T_k} + \sum_{i=1}^{\infty} \left(\alpha_{T_{k+1}-T_k-i} - \alpha_i \right) \varepsilon_{T_k-i} \neq f_{T_k-1}^{T_k} \tag{4}$$

$$\lim_{k\to\infty} E_t\left[f_{T_{k-1}}^{T_k} \right] = \mu \tag{5}$$

The proofs are given in Appendix A. Equation (3) confirms that the futures price series within a single contract will be martingale. Equation (4) reveals that a nearby futures prices series will *not* have the martingale properties, as changes in the nearby price sequence at rollover time are partially predictable. Finally, equation (5) demonstrates that the long-run expected value of a nearby futures price series equals the unconditional mean of the cash price. This characteristic is shared with any second-order stationary series: if a variable is second-order stationary then forecasts of the variables value far into the future will eventually converge to an uninformed prior which is the unconditional mean of the variable. That must be so since any shocks that explain current deviations of that variable from its unconditional mean will eventually die out. The result that the long-run forecast of the first nearby futures price series is the unconditional mean of the cash price stands in sharp contrast to characteristics of series that exhibit martingale properties.

Price dynamics in spot and futures commodity markets

The primary objective of this research is to evaluate information flows between spot and futures markets in the dairy sector. While we are ultimately interested in price discovery for milk, there is in fact no national spot market for fluid milk. As illustrated in Figure 2, farm-level milk prices are strongly linked to the prices of cheddar cheese, and the correlation between U.S. average mailbox milk price and the monthly average cheddar cheese price is 0.94.[2] Thus, a second best approach to investigating cash-futures relationships in the dairy sector seems to be to look at the cash and futures markets for cheese.[3]

The Chicago Mercantile Exchange (CME) operates a daily spot market for cheddar cheese. The cheese is traded as 40lbs blocks and 500lbs barrels, and both prices are reported daily. This market is often regarded as thin, given that only a handful of trades occur each day. Nevertheless, it is precisely this market that serves as the price discovery center for all commodity cheese in the U.S. Although cheese futures were among the first dairy futures contracts created in the early 1990s, the contracts were discontinued after the federal milk marketing order (FMMO) reform of 2000. Since 2000 there were no cheese futures available until a new cash-settled cheese futures contract started trading in July 2010. Since 2000 the central and most active dairy futures market has been for Class III milk.[4] One approach to modeling information flows between dairy spot and futures prices would be to use the cheddar cheese spot and Class III milk futures prices. However, Class III milk prices are not determined solely based on cheddar cheese prices. Per FMMO milk pricing rules, the Class III milk price is determined as a linear function of cheddar cheese, dry whey and butter prices. This induces a no-arbitrage relationship between futures prices for Class III milk, butter, dry whey and cheddar cheese. Butter futures traded throughout the 2000s, and dry whey futures started trading in 2007. Therefore, for the period 2007-2010, synthetic cheese futures can be constructed using the no-arbitrage rule as described in Bozic and Fortenbery (2012). Prior to 2007 dry whey prices were very stable. For that reason Bozic and Fortenbery (2012) use

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[2] The mailbox price is the price for fluid milk received at the farm level.

[3] Pricing of milk in the U.S. is highly regulated under Federal Milk Marketing Orders (FMMO) with federal regulations setting the minimum prices that handlers of Grade A milk must pay to farmers. The fundamental principle currently used to determine the minimum milk price is to infer the value of milk from the price of milk ingredients that have desirable nutritional qualities: casein, butterfat, and other milk solids (lactose, whey proteins, and minerals). Values of the principal milk components are inferred from national average prices for commodity dairy products: cheddar cheese, butter, dry whey and non-fat dry milk (Bozic and Fortenbery 2012).

[4] Class III milk is Grade A milk used to produce cream cheese and hard manufactured cheese.

expected dry whey prices in conjunction with the no-arbitrage relationship to create very precise approximate cheese futures for the period 2000-2007.

The spread between spot and futures cheese prices is presented in Figure 3. The cheese futures contract cash-settles against the monthly weighted average of NASS surveyed cheddar cheese prices. The CME spot cheddar cheese price serves as a price discovery tool for pricing cheese in off-exchange commercial transactions reported the following week in NASS surveys. For these two reasons the spread between spot and futures prices is minimized not at contract expiration but one month prior to contract expiry. It follows that the information flows should be examined using the 2nd nearby futures price series.

Descriptive statistics for weekly cheddar cheese spot and futures prices as well as trade data extracted from the weekly Commitments of Traders reports are given in Table 1. The data span 2000-2013.

To evaluate the time series properties of cheese cash and futures prices we employ two widely used unit root tests: the Augmented Dickey-Fuller (ADF) with automatic lag selection based on AIC criteria and the Phillips-Perron test (PP) (Said and Dickey 1984; Phillips and Perron 1988). In addition, we perform the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) stationarity test (Kwiatkowski et al. 1992). ADF and PP assume the unit root process as the null hypothesis, whereas the KPSS test uses stationarity as the null, with unit roots as the alternative hypothesis. Therefore, for a series to be judged as unit root process, ideally one would find that both ADF and PP fail to reject the null, while KPSS rejects the null.

Prior to conducting these statistical tests, prices were transformed to log-prices. Cash prices were also deseasonalized, and an exogenous structural break was imposed in January 2007 to account for the effect emerging dairy exports and rising livestock feed prices had on average prices of milk. Results of the unit root tests are presented in Table 2. The null hypothesis of unit roots is strongly rejected for cash cheese prices in both the ADF and PP tests, while KPSS fails to reject the null. This should not be very surprising. In annually harvested commodities with sufficiently high data frequency one is likely to discover shocks to be highly persistent. In contrast, cheese is produced daily. Furthermore, anecdotal evidence suggests there is excess capacity in the U.S. cheese industry so when prices of cheese rise relative to prices of dry milk products, more milk is diverted to cheese manufacturing, and supply fairly quickly compensates for most demand shocks.

What is surprising, however, is that all three tests indicate nearby cheese futures prices are also stationary. This result would seem to question market efficiency in the futures market, as a mean-reverting futures price process would suggest predictable returns to positions in futures markets.

However, the results may be a consequence of a misspecified model, as the unit root tests employed assume a linear time series process. Further insight is gained by careful examination of the regressions in the unit root tests. As explained in the previous section, when the underlying cash price series is stationary, a nearby futures price series can be nonlinear, having martingale properties within each contract segment, and mean-reverting changes at contract rollover points. Consider the ADF test for unit root autoregression with no drift. The estimated regression is

$$\Delta f_t = \zeta_1 \Delta f_{t-1} + \zeta_2 \Delta f_{t-2} + \ldots + \zeta_{p-1} \Delta f_{t-p+1} + \alpha + \rho f_{t-1} + \varepsilon_t \tag{6}$$

where $\Delta f_t = f_t - f_{t-1}$. The true process under the null hypothesis of a unit root is assumed to be the same specification as in (6) with $\alpha = 0$ and $\rho = 0$. The OLS t test for $\rho = 0$ has a non-standard distribution and critical values are the same as in Dickey-Fuller (1979) test.

Using a continuous nearby futures price series, at rollover time, $\Delta f_t = f_t^{T_k+1} - f_{t-1}^{T_k}$, where $t = T_k$, i.e. the prices being differenced are for two different delivery months. Consider a regression equivalent to (6) but with a restriction that differenced prices always use the same contract month. In particular, define $\Delta f_t = f_t^{T_k} - f_{t-1}^{T_k}$, and $f_{t-1} = f_{t-1}^{T_k}$. In Table 2. we refer to ADF tests with this refinement simply as Modified ADF tests. When rollovers are carefully treated in constructing differenced prices, the t-statistics of the lagged price variable are very small, and the null hypothesis of unit roots is never rejected.[5] Therefore, these results suggest that unit root results based on standard ADF tests may be driven by the nature of the price changes at the rollover time. In particular, this indicates that nearby futures price series are nonlinear - martingales within each contract segment, and mean-reverting at contract rollovers.

One other possible reason cheese futures may appear to be stationary using the classical unit root tests is the influence of measurement errors introduced while constructing synthetic cheese futures. As a robustness check, unit roots tests are performed for Class III milk futures, which are tightly linked to cheese futures as cheese constitutes over 80% of value of Class III milk. Results of the unit roots tests follow the same pattern as cheese futures, and modified ADF test statistics are found to be substantially smaller than ADF test statistics. As such, we can rule out the impact of measurement error on the time series properties of cheese futures.

A further analysis demonstrating the nonlinear nature of nearby futures prices is conducted by evaluating the time series properties of the cash-futures price spread, denoted $d_{j,t}$ and defined as the difference between contemporaneous average cash prices and j^{th} nearby futures prices: $d_{j,t} = c_t - f_{j,t}$. If the nearby futures were indeed represented by a unit root process, and if the cash price was truly stationary, then no linear combination of the two series could be stationary. Results reported for spreads calculated using second and fifth nearby futures demonstrate the spreads are found to be strongly stationary, further corroborating our conclusions that the nearby futures price series is in fact non-linear.

[5] Dairy futures contracts trade for delivery every month in contrast to other agricultural futures. Thus, the problems introduced by the roll-over period occurs with much higher frequency in dairy than other agricultural commodities and may explain why this has not been observed in earlier works testing for unit roots in grain and livestock futures.

Information flows between cheese futures and cash markets

When examining the information flow between cash and futures markets, it is standard practice to use the cointegration framework developed by Johansen and Juselius (1990). Bessler and Covey (1991) were among the first to introduce this method to commodity price analysis. Examples relevant for this chapter also include Fortenbery and Zapata (1997) and Thraen (1999) where co-integration was used in analyses of dairy futures and cash markets. Given the results of unit root tests presented here, it makes little sense to pursue a standard cointegration approach. Since the cash prices are clearly not an integrated process, and nearby futures price seems to be a nonlinear concatenation of unit-roots within-contract segments and mean-reverting changes at contract rollovers, a different strategy is necessary. As a result, an error-correction model is constructed with the role of the cointegrating vector taken by the spread between cash and the 2nd nearby futures price. The basic idea of cointegration is that if two integrated variables get too far apart, at least one of them will adjust to bring the variables closer together. A framework that allows the difference between cash and futures cheese prices to carry valuable forecasting information seems like a reasonable approach given the data characteristics identified earlier. Such spreads have been shown in the previous section to exhibit strong mean-reverting characteristics, so the basic model that naturally presents itself is the following:

$$\begin{aligned} \Delta c_t &= \mu_1 + \sum_{i=1}^{p} \delta_{1i} \Delta c_{t-i} + \sum_{i=1}^{p} \gamma_{1i} \Delta f_{t-i}^{j} + \theta_1 d_{t-1} + \varepsilon_{1t} \\ \Delta f_t^{j} &= \mu_2 + \sum_{i=1}^{p} \delta_{2i} \Delta c_{t-i} + \sum_{i=1}^{p} \gamma_{2i} \Delta f_{t-i}^{j} + \theta_2 d_{t-1} + \varepsilon_{2t} \end{aligned} \tag{7}$$

where $d_{t-1} = c_{t-1} - f_{t-1}^{j}$, and all futures prices are for the second nearby contract, with j being the contract month/year index of the second-nearby contract at time t. In the above model, information flow between the two markets can arise from two effects. First, short-term dynamics, captured by parameters $\delta_{ki}, \mathrm{k} = 1,2; i = 1,\dots, p$, may be important. For example, cash cheese markets trades for only ten minutes each morning, from 10:45 to 10:55am. Any news arriving after the close of the cash market, but while futures trading is still open, will affect the futures prices on day *t,* but not cash prices. Therefore, if futures markets close higher in *t* compared to *t-1*, that may have predictive power for cash price changes at trading session on date *t+1*. The other source of information flow may come from the spread between the cash and the second nearby futures price. If the spread between the cash and futures price is abnormally large, one of these two variables will have to eventually adjust and reduce the spread. If futures prices accurately anticipate the average cash price near the contract expiry, then the spread between cash and the futures price will have forecasting power in predicting changes in cash prices.

In addition to forecasting price changes, another topic of interest centers on volatility dynamics and spillovers between the two markets. The conditional variance of errors is modeled using the BEKK structure:

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N\left(0, \mathbf{H}_t\right) \tag{8}$$

$$\mathbf{H}_t = \mathbf{B}_0{}'\mathbf{B}_0 + \mathbf{A}'\varepsilon_{t-1}\varepsilon_{t-1}'\mathbf{A} + \mathbf{G}'\mathbf{H}_{\mathbf{t-1}}\mathbf{G} \tag{9}$$

Finally, the model needs to allow the examination of the impact of speculative activity on volatility levels and dynamics. A standard method to include additional regressors to BEKK variance model is to expand the structure to what is called BEKK-X:

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N\left(0, \mathbf{H}_t\right) \qquad \mathbf{H}_t = \mathbf{B}_0{}'\mathbf{B}_0 + \mathbf{A}'\varepsilon_{t-1}\varepsilon_{t-1}'\mathbf{A} + \mathbf{G}'\mathbf{H}_{\mathbf{t-1}}\mathbf{G} + \mathbf{D}'\mathbf{D}x_{t-1} \tag{10}$$

Given that the coefficients next to the additional regressors enter the variance equation in quadratic form, the signs of the coefficients for the impact on variance of both cash and futures prices are always positive. To see that, expand $D'Dx_{t-1}$ as follows

$$\mathbf{D}'\mathbf{D}x_{t-1} = \begin{bmatrix} d_{11} & 0 \\ d_{21} & d_{22} \end{bmatrix}' \begin{bmatrix} d_{11} & 0 \\ d_{21} & d_{22} \end{bmatrix} x_{t-1} = \begin{bmatrix} d_{11}^2 + d_{21}^2 & \\ d_{21}d_{22} & d_{22}^2 \end{bmatrix} x_{t-1} \tag{11}$$

It turns out that the BEKK specification can only test whether the increase in an additional regressor in the variance equation is associated with an increase in the conditional variance of cash and futures prices. As such, a hypothesis that higher speculative presence may reduce the conditional variance of futures prices cannot be easily tested with this model specification, and an alternative model structure needs to be developed.

One existing alternative to BEKK that would allow needed flexibility is a bivariate EGARCH model introduced by Nelson (1991). In the EGARCH model the logarithm of the conditional variance is modeled as a linear function of past conditional log-variances and magnitudes of realized shocks in the previous period. The exponential form allows this modeling approach to admit additional regressors in the variance equation while preserving the positive definiteness of the conditional covariance matrix. However, this model is highly nonlinear which presents estimation problems and in our case convergence could not be obtained.

The alternative presented here is the BEKK model augmented to allow more flexible treatment of additional regressors in the variance equation while preserving the positive-definiteness of the conditional covariance matrix. This is achieved by introducing the additional regressors through an exponential function that multiplies the BEKK structure:

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N\left(\mathbf{0}, \tilde{\mathbf{H}}_t\right) \qquad \tilde{\mathbf{H}}_t = \mathbf{X}_t \odot \mathbf{H}_t \tag{12}$$

where $\mathbf{H}_t$ is given by the expression in (9), the symbol $\odot$ stands for the Hadamard product, i.e. element-by-element multiplication, and the matrix X_t is defined as

$$\mathbf{X}_t = \begin{bmatrix} e^{\xi_1 \mathbf{x}_{t-1}} & \\ e^{\xi_{12} \mathbf{x}_{t-1}} & e^{\xi_2 \mathbf{x}_{t-1}} \end{bmatrix} \tag{13}$$

Due to the BEKK form, $\mathbf{H}_t$ will be positive definite, and to insure positive definiteness of $\tilde{\mathbf{H}}_t$ it will suffice to impose the following restriction on parameters ξ_{12}:

$$\xi_{12} = \frac{1}{2}(\xi_1 + \xi_2) \tag{14}$$

Denote the elements of $\mathbf{H}_t$ as

$$\mathbf{H}_t = \begin{bmatrix} \sigma_{11,t} & \\ \sigma_{12,t} & \sigma_{22,t} \end{bmatrix} \tag{15}$$

Since the exponential form is used for all elements of $\mathbf{X}_t$, the diagonal elements of $\mathbf{H}_t$ will be positive. To insure positive definiteness of $\tilde{\mathbf{H}}_t$ it will be sufficient that the determinant of the $\tilde{\mathbf{H}}_t$ matrix is positive.

$$\left|\tilde{\mathbf{H}}_t\right| = e^{\xi_1 \mathbf{x}_{t-1}} \sigma_{11,t} e^{\xi_2 \mathbf{x}_{t-1}} \sigma_{22,t} - \left(e^{\xi_{12} \mathbf{x}_{t-1}} \sigma_{22,t}\right)^2 > 0 \tag{16}$$

With restriction (14) this is reduced to

$$\left|\tilde{\mathbf{H}}_t\right| = e^{\xi_1 \mathbf{x}_{t-1} + \xi_2 \mathbf{x}_{t-1}} \left[\sigma_{11,t} \sigma_{22,t} - \left(\sigma_{22,t}\right)^2\right] \tag{17}$$

The positive range of the exponential function together with positive-definiteness of $\mathbf{H}_t$ imposed by the BEKK structure jointly guarantees that $\tilde{\mathbf{H}}_t$ will be positive-definite. In practice, we recommend starting with the unrestricted version given in (13), and after the model is estimated checking for positive-definiteness of the conditional covariance matrix for each observation. If conditional covariance matrices are not positive definite for all time points, the restriction stated in (14) will resolve the problem. The new model is referred to as BEKK-MEX, where MEX stands for multiplicative exponential heteroskedasticity.

BEKK-MEX has four important characteristics. First, the covariance matrix is always positive definite, as demonstrated in (17). Second, since the exponential function is always positive, signs of coefficients $\boldsymbol{\xi}_1, \boldsymbol{\xi}_2$ do not need to be restricted as in (11). Third, if additional regressors do not explain volatility, i.e. $\boldsymbol{\xi}_1 = \boldsymbol{\xi}_{12} = \boldsymbol{\xi}_2 = \mathbf{0}$, the model collapses to a standard BEKK model. Finally, with restriction (14) additional regressors impact only conditional variances of individual series, but not conditional correlation directly:

$$\rho_t = \frac{\tilde{\sigma}_{12,t}}{\tilde{\sigma}_{11,t}\tilde{\sigma}_{12,t}} = \frac{\sigma_{12,t} e^{\frac{1}{2}(\boldsymbol{\xi}_1+\boldsymbol{\xi}_2)\mathbf{x}_{t-1}}}{\sigma_{11,t} e^{\frac{1}{2}\boldsymbol{\xi}_1\mathbf{x}_{t-1}} \sigma_{22,t} e^{\frac{1}{2}\boldsymbol{\xi}_2\mathbf{x}_{t-1}}} = \frac{\sigma_{12,t}}{\sigma_{11,t}\sigma_{22,t}} \tag{18}$$

However, conditional correlation is time-varying, and influenced by additional regressors indirectly, through impacts on lagged conditional variances that enter the BEKK structure.

The complete model for evaluating information flows between cash and futures prices, and the influence of speculators on both price levels and volatility dynamics is as follows:

$$\begin{aligned} \Delta c_t &= \mu_1 + \delta_{11}\Delta c_{t-1} + \ldots + \delta_{14}\Delta c_{t-4} + \gamma_{11}\Delta f_{t-1}^i + \ldots \gamma_{14}\Delta f_{t-4}^i + \theta_1 d_{t-1} + \varepsilon_{1t} \\ \Delta f_t^i &= \mu_2 + \delta_{21}\Delta c_{t-1} + \ldots + \delta_{24}\Delta c_{t-4} + \gamma_{21}\Delta f_{t-1}^i + \ldots \gamma_{24}\Delta f_{t-4}^i + \varphi SPEC_{t-1} + \theta_2 d_{t-1} + \varepsilon_{2t} \end{aligned} \tag{19}$$

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N\left(0, \mathbf{H}_t\right)$$

$$\tilde{\mathbf{H}}_t = \mathbf{X}_t \odot \mathbf{H}_t \tag{20}$$

$$\mathbf{H}_t = \mathbf{B}_0{}'\mathbf{B}_0 + \mathbf{A}'\varepsilon_{t-1}\varepsilon_{t-1}'\mathbf{A} + \mathbf{G}'\mathbf{H}_{t-1}\mathbf{G}$$

$$\mathbf{X}_t = \begin{bmatrix} e^{\xi_1 c_{t-1} + \phi_1 w_{t-1}} & \\ e^{\xi_{12} c_{t-1} + \phi_{12} w_{t-1}} & e^{\xi_2 c_{t-1} + \phi_2 w_{t-1}} \end{bmatrix} \tag{21}$$

In the MEX matrix, we have used lagged cash prices in addition to a measure of speculative adequacy, to control for possible confounding if speculative activity coincides with cycles in milk prices and volatility increases in price levels.

A classical measure of speculative adequacy is called Working's T, and was introduced by Working (1960). Working's T measures the 'adequacy' of speculation. The minimum value is equal to 1, and the index takes that value when speculative positions just suffice to offset net commercial positions. Markets with T-index less than 1.15 are considered to have insufficient liquidity (Irwin and Sanders, 2010).

Denote commercial long (short) positions with $H_L \left(H_S\right)$ and noncommercial, (i.e. speculative) long (short) positions by $S_L \left(S_S\right)$, all measured by the number of contracts held. When short hedging exceeds long hedging, Working's T is calculated as

$$T = \frac{S_L + 2H_L}{H_S + H_L} = 1 + \frac{S_S}{H_S + H_L} \tag{22}$$

If the hedging position is net long, (i.e. $H_L > H_S$) then the formula becomes

$$T = 1 + \frac{S_L}{H_S + H_L} \tag{23}$$

To calculate T, all open interest has to be allocated to these four categories (H_L, H_S, S_S, S_L). That means that nonreportable positions, for which no information is available relative to their speculative or hedging nature, have to be allocated to the above categories. Following Rutledge (1977) and Irwin and Sanders (2010) nonreportable positions are allocated to the commercial and noncommercial categories in the same proportion as that which is observed for reporting traders. For robustness, alternative model specifications similar to Peck (1980) are estimated where nonreportable positions are allocated to obtain either upper or lower bounds of Working's T index (in other words, all nonreportables are treated either as hedgers or speculators). While Working's T is used for the volatility model, the net long speculative position, as a percent of total open interest, is used as a measure of speculative pressure on futures price levels in equation (9).

Results and discussion

The results of the model (19)-(21) are presented in Tables 3 and 4. Since the conditional covariance matrix is found positive for each time period, it is not necessary to impose equation (14) as a restiriction.[6]

Coefficients in the mean equation indicate bi-directional Granger causality. First, cash-futures spread has strong predictive power in anticipating later cash prices. The negative coefficient ϕ_1 indicates that when cash prices are higher than futures prices, cash prices in the next period are forecast to decline. In addition, the previous week's changes in futures prices are predictive of cash price changes. The coefficient ϕ_2 in the futures price equation is also statistically significant, though six times lower in absolute value than ϕ_1 indicating that futures markets are the primary center of price discovery. However, this result may be an artefact of utilized data frequency. As stated above, daily spot trading in cheese is closed at 10:55am, while end-of-day futures prices are collected at 1:10pm. Anecdotal evidence offered by dairy traders suggests that the cash market does play an important role in price discovery. Further research using the intraday futures prices is needed to examine that claim. Net long speculative positions are not found to be predictive of futures price changes.

[6] To conserve space, the results for the five alternative specifications estimated for robustness checks are available from the authors.

Elements of the **A** and **G** matrices capture volatility spillovers between cash and futures markets. In particular, if $a_{21} = 0, g_{21} = 0$ then past futures price shocks and conditional variances do not help forecast the cash price conditional variance. Likewise, if $a_{12} = 0$ and $g_{12} = 0$ then past cash price shocks and conditional variances do not have forecasting power for the futures price conditional variance. In Table 4 all four of these coefficients are statistically significant, indicating bidirectional volatility spillovers.

The evidence of speculative activity, as measured by Working's T, on cash and futures price volatility is somewhat mixed. When models are estimated using the restriction from equation (14) the coefficients indicate that a higher Working's T index predicts a higher conditional variance for futures prices. Results are compared in Table 5. However, conditional covariance matrices are positive definite in all periods even without imposing equation (14). Furthermore, restricted models are regularly rejected using the likelihood ratio tests. Therefore, we suggest that the unrestricted model is the more appropriate model specification. Within that specification, we find robust evidence that higher speculative presence is associated with lower conditional variances of futures prices. Working's T calculated using the proportional method suggested by Irwin and Sanders (2010) reveals an average Working's T index of only 1.03, much lower than any of the twelve agricultural futures markets analyzed by Irwin and Sanders (2010). Estimated results and descriptive statistics on Working's T jointly suggest dairy futures markets would benefit from stronger speculative presence which would increase market liquidity.

Conclusions

The purpose of this paper is to evaluate price discovery, volatility spillovers and impacts of speculation in the U.S. dairy sector. In order to appropriately model the information flow between cash and futures markets, it is important to understand the time series properties of both cash and futures prices. This article contributes to the debate on presence of unit roots in commodity prices by pointing out that production theory, which predicts cash commodity prices will be mean-reverting when the production function exhibits constant returns to scale, can be reconciled with the efficient market hypothesis which is consistent with unit root process for futures prices. In a theoretical analysis we demonstrate that when the underlying cash price series does not have a unit root, nearby futures price series can be nonlinear, exhibiting a martingale process within each contract segment, and mean-reverting changes at contract rollover points. While this theoretical result will not map easily to annually harvested commodities, we do find evidence that cash and futures dairy prices exhibit properties suggested by this analysis.

To deal with the data issues, a new ECM-BEKK-MEX model is developed which includes an error-correction model for price levels and an augmented GARCH-BEKK variance model. Unlike regular BEKK models which impose severe restrictions on the use of additional regressors in the variance equation, the BEKK-MEX model can handle additional regressors very flexibly, while preserving positive definiteness of the conditional variance matrix. The ECM-BEKK-MEX model developed is utilized in an analysis of cheddar cheese cash and futures markets. Results suggest futures markets represent the primary center for price discovery, although both information flows and volatility spillovers are bidirectional. Net long speculative positions do not forecast the direction of change of futures prices, but more speculation, as measured by Working's T index, is associated with a lower conditional variance of futures prices in the next period. The results may challenge beliefs popular among many policymakers, but are in fact consistent with the recent literature on the role of speculators in agricultural commodity markets.

Future research is planned to address the price discovery role of spot CME dairy markets using high-frequency futures price data. Because of the differences daily in trading periods between cash and futures markets for dairy, use intra-day futures would provide a better understanding of the role of cash prices in futures price discovery.

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Table 1. U.S. Dairy Markets 2000-2013 – Descriptive Statistics

	No. Obs.	Average	St. Dev.	Minimum	Maximum
Cheddar Cheese					
Spot Price	652	1.52	0.28	1.05	2.24
2nd Nearby Futures	652	1.54	0.26	1.09	2.23
Class III Milk					
Open Interest	652	36,986	12,688	11,352	63,336
Commercial Long Position (% of OI)	652	62.20%	8.88%	35.92%	38.59%
Commercial Short Position (% of OI)	652	54.76%	8.36%	82.45%	76.04%
Noncommercial Long Position (% of OI)	652	7.97%	5.11%	<0.01%	33.26%
Noncommercial Short Position (% of OI)	652	6.64%	3.73%	<0.01%	16.83%
Non-reportable Long Position (% of OI)	652	13.87%	3.90%	6.97%	28.83%
Non-reportable Short Position (% of OI)	652	22.63%	5.81%	12.30%	24.69%

Note: To make cheddar cheese spot and futures prices comparable, cheddar cheese spot price is created as a simple average of spot prices for 40lbs blocks and 500lbs barrels price augmented by 3 cents.

Table 2: Unit root tests

	Augmented Dickey-Fuller	Phillips-Perron	Modified Aug. Dickey-Fuller.
Cash Cheddar Cheese	-3.99***	-4.08***	-3.99**
Cheese Futures: 1st nearby	-2.77*	-3.13**	2.46
Cheese Futures: 2nd nearby	-3.03**	-2.95**	-0.55
Cheese Futures: 3rd nearby	-2.88**	-2.89**	-0.65
Cheese Futures: 4th nearby	-2.50	-2.71*	-1.26
Cheese Cash – Futures 1st nearby	-10.11***	-10.54***	-5.42**
Cheese Cash – Futures 2nd nearby	-3.90***	-12.46***	-17.24***
Cheese Cash – Futures 3rd nearby	-6.38***	-7.08***	-10.61***
Cheese Cash – Futures 4nd nearby	-4.46***	-5.52***	-7.73***
Class III Milk Futures: 1st nearby	-2.59*	-2.82*	1.88
Class III Milk Futures: 2nd nearby	-2.60*	-2.53	-0.20
Class III Milk Futures: 3rd nearby	-2.47	-2.48	-0.25
Class III Milk Futures: 4th nearby	-2.50	-2.42	-0.80

Note: Reported numbers are t-statistics used for unit-root tests. Significance at 10%, 5% and 1% is indicated with one, two, and three stars respectively. For augmented Dickey-Fuller (ADF) and Phillips-Perron tests optimal lag length is determined using AIC criteria. For cash prices, deseasonalized series is used in tests.

Table 3. Estimation Results – Price Level Model

Variable	Cash Price Equation	Futures Price Equation
Δc_{t-1}	0.04	<0.01
	(0.03)	(0.03)
Δc_{t-2}	0.03	-0.01
	(0.04)	(0.03)
Δc_{t-3}	-0.02	<0.01
	(0.04)	(0.03)
Δc_{t-4}	-0.02	-0.07***
	(0.03)	(0.03)
Δf_{t-1}	0.31***	0.02
	(0.05)	(0.04)
Δf_{t-2}	0.13***	0.06
	(0.04)	(0.04)
Δf_{t-3}	0.10**	0.08**
	(0.04)	(0.04)
Δf_{t-4}	0.05	0.03
	(0.04)	(0.04)
$c_{t-1} - f_{t-1}$	-0.31***	0.05**
	(0.02)	(0.02)

Note: 10%, 5% and 1% confidence levels marked with *, **, and *** respectively.

Table 4. Estimation Results – Volatility Model

BEKK		MEX	
Parameter	Estimate	Parameter	Estimate
b_{11}	0.002 (0.002)	ξ_1	0.06 (0.05)
b_{21}	-0.034 (0.034)	ξ_{12}	-0.06 (0.08)
b_{22}	-0.048*** (0.012)	ξ_2	0.49*** (0.13)
a_{11}	0.258*** (0.045)	ϕ_1	0.30 (0.19)
a_{12}	-1.039*** (0.349)	ϕ_{12}	-1.45*** (0.21)
a_{21}	-0.382*** (0.055)	ϕ_2	-3.39*** (0.37)
a_{22}	1.674*** (0.478)		
g_{11}	0.699*** (0.065)		
g_{12}	1.308*** (0.486)		
g_{21}	0.164*** (0.056)		
g_{22}	3.568*** (0.496)		

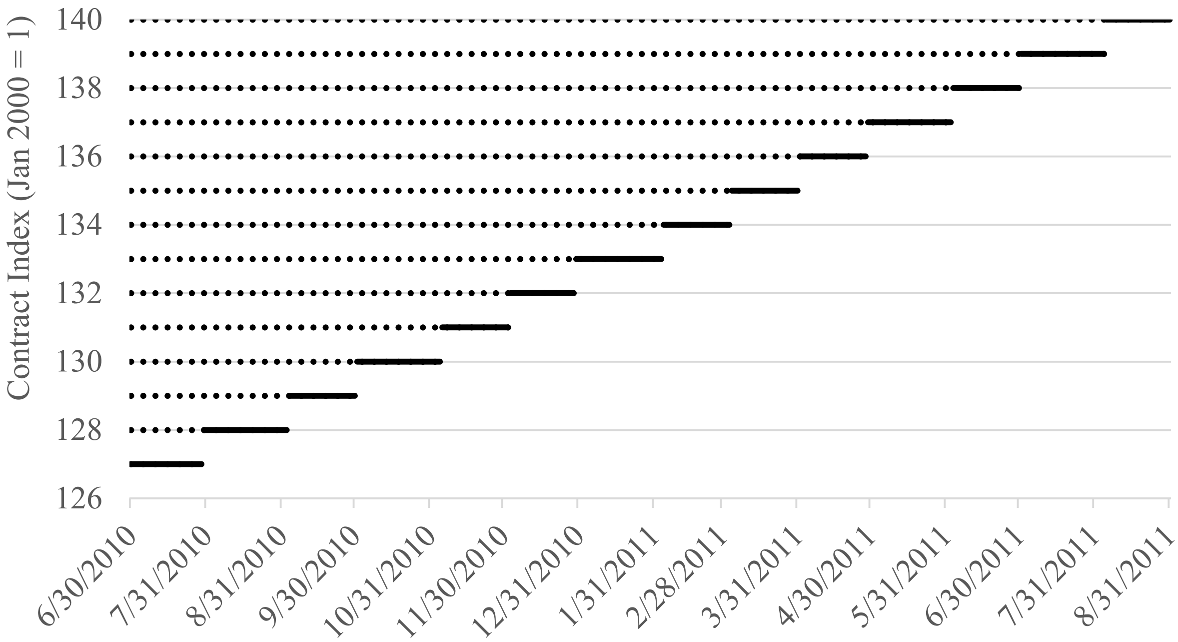
Note: 10%, 5% and 1% confidence levels marked with *, **, and *** respectively.

Table 5. The impact of Excess Speculation on Conditional Variance of Futures Prices

Working's T Index Calculation Method	Avg.	Min	Max	Unrestricted correlation Log-like.	Unrestricted correlation Coeff.	Restricted correlation Log-like.	Restricted correlation Coeff.	LR test§
Proportional	1.03	1.00	1.08	2890.88	-3.39*** (0.37)	2879.76	0.19** (0.08)	22.24***
Upper bound	1.16	1.06	1.29	2892.35	-1.14*** (0.34)	2886.19	0.19*** (0.03)	12.32***
Lower bound	1.02	1.00	1.09	2887.65	-1.46*** (0.71)	2882.74	0.23*** (0.09)	9.82***

Note: Presented coefficients are denoted with ϕ_2 in equation (21).

Figure 1. Constructing nearby futures price series



Note: Solid line segments are contatenated to create the 1st nearby futures price series.

Figure 2. US. Mailbox Milk Price vs. Monthly Average Cheddar Cheese Price

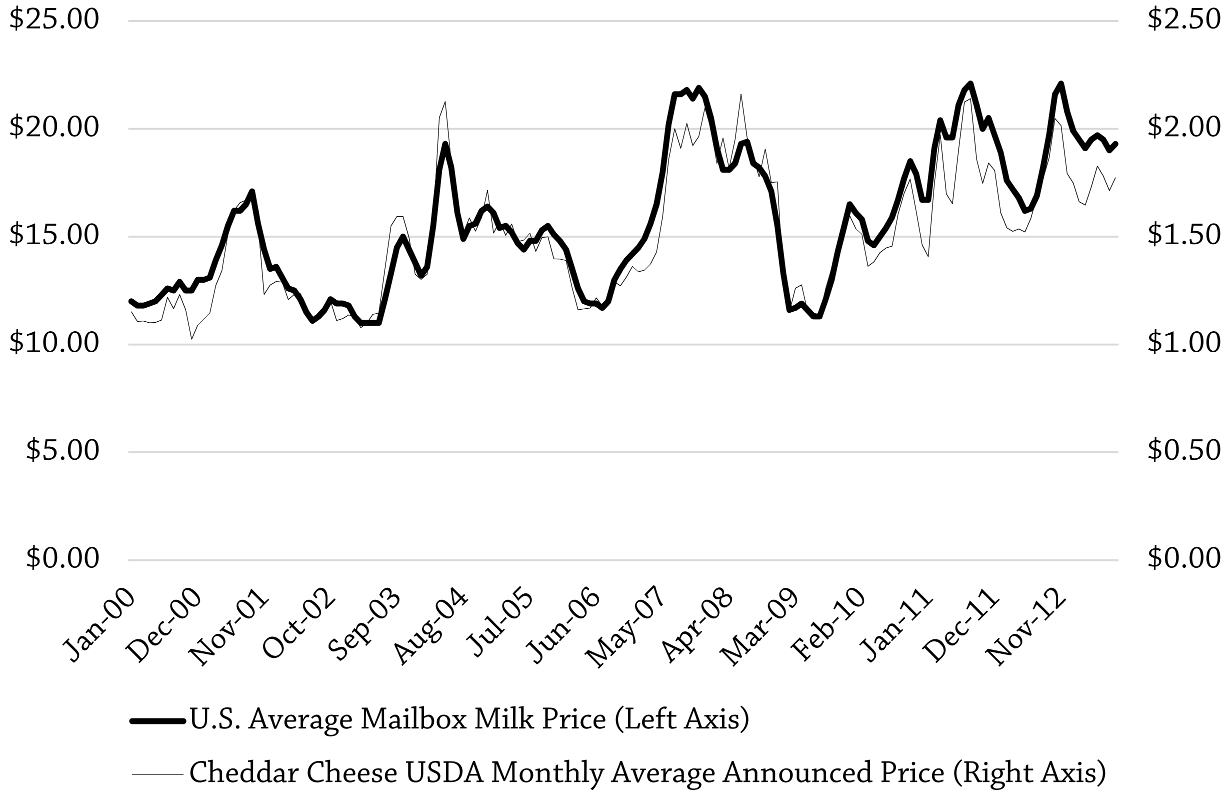
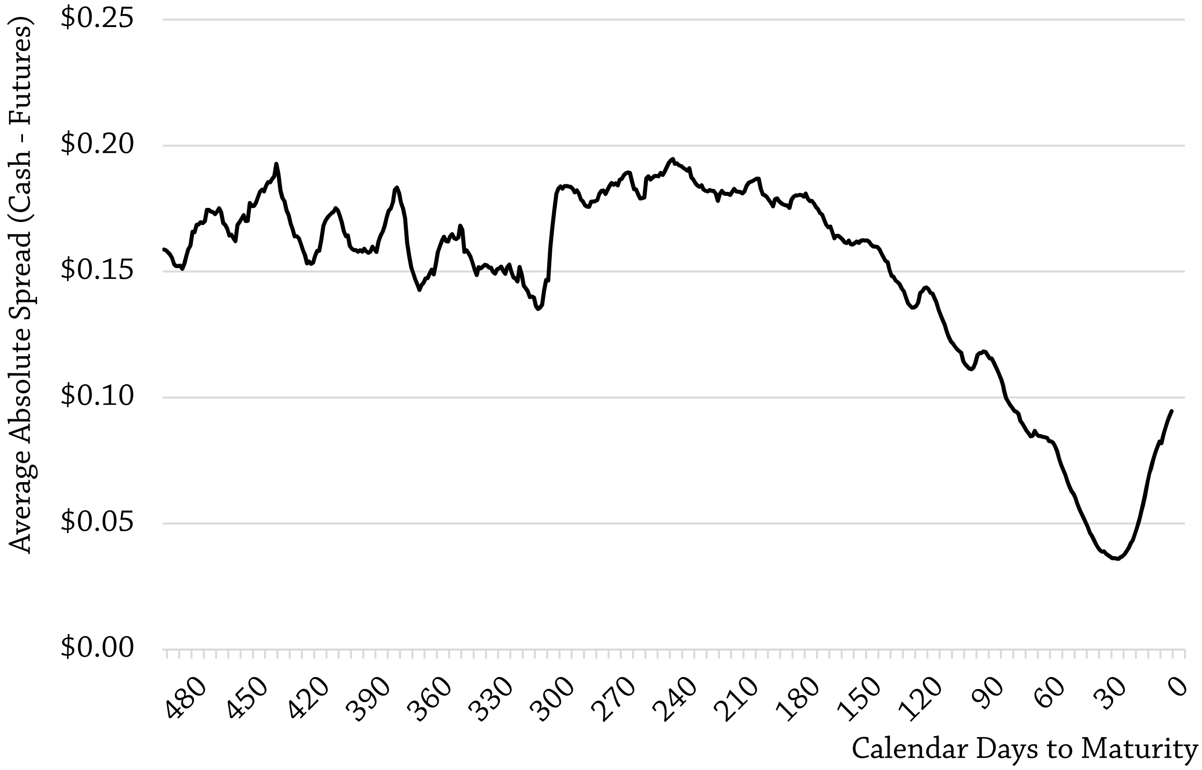


Figure 3. Average absolute value of cheese cash-futures spread, as a function of time to maturity.



Appendix A. Time Series Properties of a Nearby Futures Price Series when Cash Price is Stationary

Suppose a cash price series for some commodity is second-order stationary. Suppose further that a futures contract is written for that commodity, and for simplicity, assume that there is no basis at futures contract expiry, i.e. the terminal futures price equals the cash price at the contract expiration. Finally, assume that futures prices are efficient and embody no risk premium. This appendix contains the analysis of time series properties of an *n-th* nearby futures price series when the above assumptions hold.

Let μ be the unconditional mean of the cash price, and σ_c^2 be the unconditional variance. By the Wold decomposition theorem (Wold, 1954), there exists the unique fundamental moving average representation of the cash price stochastic process:

$$c_t = \mu + \sum_{i=0}^{\infty} \alpha_i \varepsilon_{t-i} \tag{A.1}$$

where $\alpha_0 = 1, E(\varepsilon_t) = 0, \mathrm{E}(\varepsilon_t \varepsilon_{t-k}) = 0, \forall \mathrm{k}$. Denote futures price at time t for a contract that expires at time T by f_t^T. Efficient futures prices that do not incorporate risk premiums will be unbiased predictors of cash prices at contract expiry:

$$f_t^T = E_t[c_T] \tag{A.2}$$

Using Wiener-Kolmogorov prediction formula (Hansen and Sargent, 1980) futures prices at time t can be expressed as

$$f_t^T = \mu + \left(\frac{\alpha(L)}{L^{T-t}} \right)_+ \varepsilon_t = \mu + \sum_{i=0}^{\infty} \alpha_{T-t+i} \varepsilon_{t-i} \tag{A.3}$$

where the annihilation operator $(\ )_+$ replaces all negative lag values by zero. An alternative, and equivalent expression for (A.3) is

$$f_t^T = \mu + \sum_{i=T-t}^{\infty} \alpha_i \varepsilon_{T-i} \tag{A.4}$$

Exploiting notation in (A.4) it will be shown that under the assumptions of this model, any change to futures prices of a single contract must come from unanticipated information shocks ε_{t+1}. First, f_{t+1}^T are expressed using Wiener-Kolmogorov formula as

$$f_{t+1}^{T} = \mu + \sum_{i=T-(t+1)}^{\infty} \alpha_i \varepsilon_{T-i} = \mu + \sum_{i=T-t}^{\infty} \alpha_i \varepsilon_{T-i} + \alpha_{T-(t+1)} \varepsilon_{t+1} = f_t^{T} + \alpha_{T-t-1} \varepsilon_{t+1} \tag{A.5}$$

Since $E_t\left(\varepsilon_{t+1}\right) = 0$ it follows that $E_t\left[f_{t+1}^{T}\right] = f_t^{T}$ and the martingale property of futures prices of the same contract is established. If in addition fundamental moving average coefficients increase in absolute value as their index decreases (e.g. this would be the case for AR(1) models) then the conditional variance of futures prices

$$Var_t\left[f_{t+1}^{T}\right] = \alpha_{T-t-1}^{2} \sigma_c^{2} \tag{A.6}$$

will be increasing as time to maturity decreases. This is the well-known “Samuelson Effect“ (Samuelson, 1965).

From the analysis undertaken above, it would be wrong, however, to conclude that because prices within a single futures contract are martingales that an n-th nearby futures (that is a single time series of prices constructed across different maturities) price series will necessarily exhibit the same property. Let the first nearby futures price series be constructed by rolling contracts over one day before the delivery date:

$$F_1 = \left(f_1^{T_1}, \ldots, f_{T_1-1}^{T_1}, f_{T_1}^{T_2}, \ldots f_{T_2-1}^{T_2}, f_{T_2}^{T_3}, \ldots \right) \tag{A.7}$$

It will help to take a closer look at the MA representation of futures prices around the rollover date:

$$\begin{aligned} f_{T_k-1}^{T_k} &= E_{T_k-1}\left[c_{T_k}\right] = \mu + \sum_{i=1}^{\infty} \alpha_i \varepsilon_{T_k-i} \\ f_{T_k}^{T_{k+1}} &= E_{T_k}\left[c_{T_{k+1}}\right] = \mu + \sum_{i=T_{k+1}-T_k}^{\infty} \alpha_i \varepsilon_{T_{k+1}-i} \end{aligned} \tag{A.8}$$

The difference in consecutive futures prices of this nearby series at rollover time is

$$f_{T_k}^{T_{k+1}} - f_{T_k-1}^{T_k} = \alpha_{T_{k+1}-T_k} \varepsilon_{T_k} + \sum_{i=1}^{\infty} \left(\alpha_{T_{k+1}-T_k-i} - \alpha_i \right) \varepsilon_{T_k-i} \tag{A.9}$$

Only the first part of the difference, $\alpha_{T_{k+1}-T_k} \varepsilon_{T_k}$, is not known at time $T_k - 1$, while the second part, the infinite sum, is fully known at that time. It follows that

$$E_{T_k-1}\left[f_{T_k}^{T_{k+1}}\right] = f_{T_k-1}^{T_k} + \sum_{i=1}^{\infty} \left(\alpha_{T_{k+1}-T_k-i} - \alpha_i \right) \varepsilon_{T_k-i} \neq f_{T_k-1}^{T_k} \tag{A.10}$$

The first nearby futures price series will *not* have the martingale properties, and changes in the nearby price sequence at rollover time are partially predictable. To give a simple example, suppose that current first nearby contract is the March contract, and tomorrow the first nearby contract will be the futures price for delivery in April, i.e. rollover is to occur tomorrow. Then the expected change in the first nearby price series is the simple difference between today's futures price for April delivery and today's futures price for March delivery.

Further insight can be extracted from (A.10). If it so happens that the cash price at time $T_k - 1$, $c_{T_{k-1}} = \mu + \sum_{i=1}^{\infty} \alpha_i \varepsilon_{T_k - i}$ is above the long run mean μ, then the sum $\sum_{i=1}^{\infty} \alpha_i \varepsilon_{T_k - i}$ will be positive. When fundamental moving average coefficients are monotonically declining in absolute value, i.e. $i < j \Rightarrow |a_i| > |\alpha_j|, \forall i, j \geq 0$ then the infinite sums from expression (A.8) can be ordered in absolute value:

$$\left| \sum_{i=T_{k+1}-T_k}^{\infty} \alpha_i \varepsilon_{T_{k+1}-i} \right| < \left| \sum_{i=1}^{\infty} \alpha_i \varepsilon_{T_k - i} \right| \tag{A.11}$$

If condition (A.11) holds, and $c_{T_{k-1}} > \mu$ then $\sum_{i=1}^{\infty} \left(\alpha_{T_{k+1}-T_k-i} - \alpha_i \right) \varepsilon_{T_k - i} < 0$. In other words, the predictable component in the first nearby price change at contract rollover will be mean-reverting. In addition, because cash price is assumed to be second-order stationary, moving average coefficients are square summable, i.e. $\sum_{i=0}^{\infty} \alpha_i^2 < \infty$. This implies that

$$\lim_{i \to \infty} \alpha_i = 0 \tag{A.12}$$

Since for a fixed t, $k \to \infty \Rightarrow T_k - t \to \infty$ it follows that

$$\begin{aligned} \lim_{k\to\infty} E_t\left[f_{T_{k-1}}^{T_k} \right] &= \lim_{k\to\infty} E_t\left[E_{T_k - 1}\left[c_{T_k} \right] \right] = \lim_{k\to\infty} E_t\left[c_{T_k} \right] \\ &= \lim_{k\to\infty} E_t\left[\mu + \sum_{j=0}^{\infty} \alpha_j \varepsilon_{T_k - j} \right] = \lim_{k\to\infty}\left[\mu + \sum_{i=o}^{\infty} \alpha_{T_k - t + j} \varepsilon_{t-j} \right] = \mu \end{aligned} \tag{A.13}$$

In words, the long-run expected value of the first nearby futures price series is the unconditional mean of the cash price. This characteristic is shared with any second-order stationary series: if a variable is second-order stationary then forecasts of the variables value far into the future will eventually converge to an uninformed prior which is the unconditional mean of the variable. That must be so since any shocks that explain current deviations of that variable from its unconditional mean will eventually die out. The result that the long-run forecast of the first nearby futures price series is the unconditional mean of the cash price stands in sharp contrast to characteristics of series that exhibit martingale properties. For such a series, $\lim_{k\to\infty} E_t\left[x_{t+k}\right] = x_t$, i.e. all shocks are permanent, and the long-run forecast is equal to the last observed value of the variable.

The argument that the first nearby price series will be mean-reverting at contract rollover carries forward to the n-th nearby series, which is demonstrated by comparing the first and an n-th nearby price series at rollover. For the first nearby series, price change at rollover time is given in (A.9). Since futures prices are assumed unbiased predictors of future cash prices, (A.9) can be rewritten as

$$f_{T_k}^{T_{k+1}} - f_{T_k-1}^{T_k} = E_{T_k}\left[c_{T_{k+1}}\right] - E_{T_k-1}\left[c_{T_k}\right] \tag{A.14}$$

Similarly, for the n-th nearby price series, $f_{T_k}^{T_{k+n}} - f_{T_k-1}^{T_{k+n-1}} = E_{T_k}\left[c_{T_{k+n}}\right] - E_{T_k-1}\left[c_{T_{k+n-1}}\right]$. From (A.12) it follows that

$$\lim_{n\to\infty} f_{T_k}^{T_{k+n}} - f_{T_k-1}^{T_{k+n-1}} = 0 \tag{A.15}$$

In other words, the predictabile part of price change for n-th nearby contract at rollover time will be smaller the higher the *n* is. Mean-reverting changes at contract rollover will be most pronounced in the first nearby, less so in the second nearby, and even less in the third nearby price series, etc.

In conclusion, it is shown that when the cash price series is second-order stationary, futures prices for a specific contract will be a martingale, but not a random walk, as random walk assumes constant variance of shocks. In contrast, we expect to see the Samuelson effect, i.e. increases in futures price volatility as time to maturity declines. Furthermore, the n-th nearby futures series will be nonlinear, having martingale properties within each contract segment, and mean-reverting changes at contract rollover. When fundamental MA coefficients are monotonically declining in absolute value, mean-reverting change at contract rollover will be less pronounced for further horizon series.