Predictive Formulas for Yield of Cheese from Composition of Milk: A Review[1]

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ABSTRACT

Various yield formulas are described or developed where cheese is considered as a three-phase system of fat, para-casein, and water and water solubles. Type A formulas distribute moisture, whey solids, and salt proportionally to both para-casein and fat in cheese. Type B formulas include whey solids and salt with the para-casein and distribute moisture proportionally to fat and fat-free cheese. Type C formulas include whey solids, salt, and moisture only with para-casein. Type E formulas are those based on actual cheese making. Types A, B, and C formulas were developed from the basic yield formula of yield equaling recovered fat plus complex of recovered para-casein and calcium phosphate plus cheese whey solids plus cheese moisture. It would appear that they could be applied to most varieties of cheese. However, research is needed to verify constants in predictive formulas under commercial conditions.

The formulas include whey solids as a separate factor, which is necessary when moisture in cheese varies. The formulas were adapted to include a "solute-exclusion" factor for that portion of moisture bound to para-casein that does not contain whey solids.

The merits of targets of constant moisture in cheese versus constant moisture in the fat-free cheese are discussed; the latter is desirable for quality and for sensory considerations when the casein:fat ratio in milk is not constant, particularly for reduced fat variants of cheese varieties. Type A and type B formulas use moisture; those of type C use moisture in fat-free cheese.

Predictive yield formulas from milk composition are discussed for application in industrial or experimental cheese making. They can serve as targets for yield, as a base in expressing actual yields as percentage of theoretical yield, and for application in multiple component pricing of milk.

INTRODUCTION

There has been interest in relating the yield of cheese to components in milk since the last century. Van Slyke (48), Babcock (3), Shuttleworth (45), and probably others correctly related yield of cheese to the amount of fat and casein in milk. Out of the work in New York state arose the well-known formula of Van Slyke and Price (51)(VSP) published originally by Van Slyke and Publow (52); Babcock (3) also published a formula. In other classical work, McDowall (37) observed a different relationship between milk fat, casein, and yield of Cheddar cheese in New Zealand. Posthumus et al. (42) (PBK) developed

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a detailed formula for the yield of Dutch-type cheese; Lolkema (30, 31) described practical formulas for the same cheese; these formulas could be applied to other cheese varieties, such as Cheddar, by changing constants.

Yield is of basic importance to the cheese industry. Small differences in yield translate to large sums of money for cheese plants. On a national scale, a yield difference of .1% for cheese, worth $5.00/kg, makes a difference of $1,250,000 annually in Canada, and about 10 times that in the United States.

Sophisticated yield formulas are used successfully in The Netherlands to help control moisture content, cheese yield, and cheesemaking efficiency (30, 42). If actual yield is larger or smaller than predicted, this indicates higher or lower moisture content than is desired or legal, signaling a change in manufacturing procedure. Not all, however, advocate the use of predictive yield formulas, preferring to control cheese making by monitoring critical losses and components of cheese (20, 40). For those who have not been privy to what has led to those divergent conclusions, it is useful to examine both systems critically. As a first step in this examination, a review is of yield formulas is necessary.

The purpose of this paper is to present some new "general" cheese yield formulas as well as to review some established formulas. The paper examines their interrelationships and their characteristics relative to certain applications. This is relevant to other studies on the effect of enzymes and other treatments on cheese yield, on multiple component pricing of milk, and on the control of industrial cheese making.

Considerable material is in appendices. It is intended, however, that the main text should be readable by itself with reference to the Appendices only for more detailed explanations if the reader wishes, except for Appendix 1, which contains terms and abbreviations used in this paper. This amalgam of abbreviations describes various terms from other authors, since no one system could be used. Hence, some quoted formulas are not exactly as originally described. The other appendices are to assist in understanding the derivations of the various formulas by those who wish to modify, adapt, or compare them.

General Considerations

Yield formulas can be grouped into two general classes, those based on a target composition of cheese (types A, B, C, and D) and those derived from actual yield of cheese from milk of varied composition (type E).

In the first class, there are at least four general types of formulas. These assume that cheese consists of three phases — fat phase, para-casein-network phase, and water-soluble phase — with the last consisting of water and soluble solids. These three phases are clearly shown in Figure 1. Figure 1a is a scanning electron micrograph showing the globular fat as a discontinuous phase in the continuous fat-free phase. Figure 1b is a scanning electron micrograph of fat-free cottage cheese showing the para-casein network and the interstitial water phase, both continuous phases.

The four ways of looking at cheese are illustrated in Figure 2. In Figure 2A, whey solids, salt, and moisture are distributed proportionally to fat and para-casein (type A formulas). In the second (B), whey solids and salt are included with the para-casein to form fat-free dry cheese, and moisture is distributed proportionally to fat and to fat-free dry cheese (type B formulas). The terms "fat-free" and "fat-free dry" cheese are used frequently; the latter is the complex of para-casein and calcium phosphate, plus the whey solids plus the salt, i.e., the cheese minus the fat and moisture. In the third (C), moisture, salt, and whey solids are distributed only to para-casein (type C formulas). In the fourth (D), salt, whey solids, and moisture are treated together as a water phase and all phases compared on a volume basis; partial volumes of .9, 1.0, and 1.6 g/ml were used for fat, water, and other components, respectively (type D formulas). The concept of cheese in Figure 1 indicates that the water phase belongs with the para-casein as in formula types C and D and not as indicated in A and B.

These pictures of cheese components are compatible with a general formula:

Yield = cheese fat + complex of para-casein and CaH_2PO_4 in cheese + cheese salt + whey solids in cheese + cheese moisture, [1]

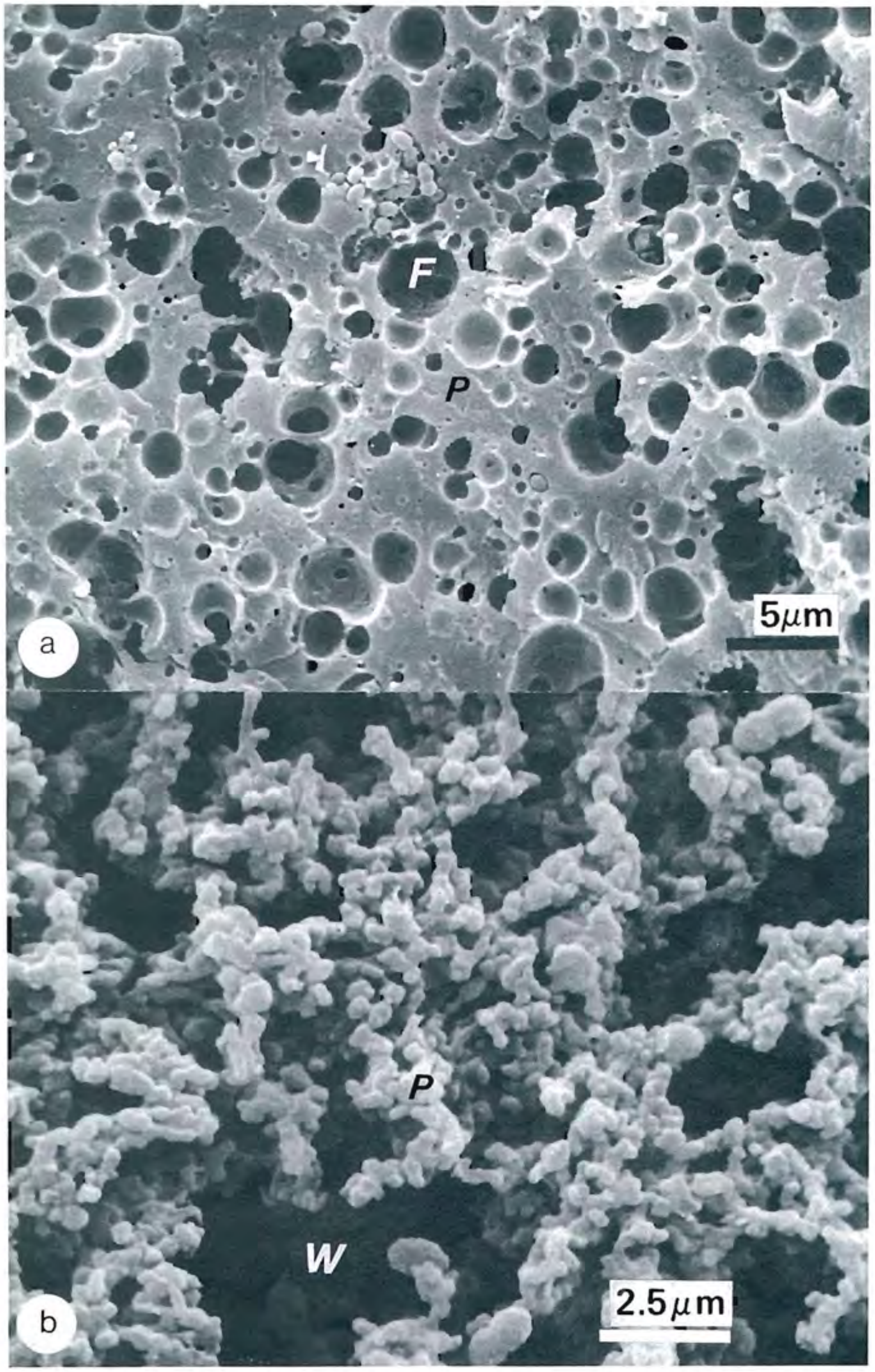

Figure 1. The three-phase nature of cheese: fat, casein, and water-soluble components. Scanning electron micrograph (A) of Cheddar cheese showing discontinuous phase fat globules (F) and continuous phase fat-free cheese (P). Scanning electron micrograph (B) of fat-free cottage cheese showing strands of casein with interstitial water-soluble materials (W), both continuous phases. (Courtesy of M. Kalab).

which can be written:

$$Y = FK_f + CK_c + CS + CWS + CM \qquad [2]$$

In theory, they can be expressed on a weight or volume basis; weight is used this paper. It should be noted, for example, that cheese salt is not the level of salt in cheese but rather the amount of salt in cheese from 100 kg of milk and equals the level of salt times yield.

The other general class of yield formulas is termed type E; these formulas are derived from actual cheese making under relatively constant conditions to produce cheese of quality as uniform as possible.

Table 1 lists general formulas of types A, B, and C. Their derivations are described in Appendix 3 from the general Formulas 1 and 2. Table 2 lists formulas of types A, B, C, and E

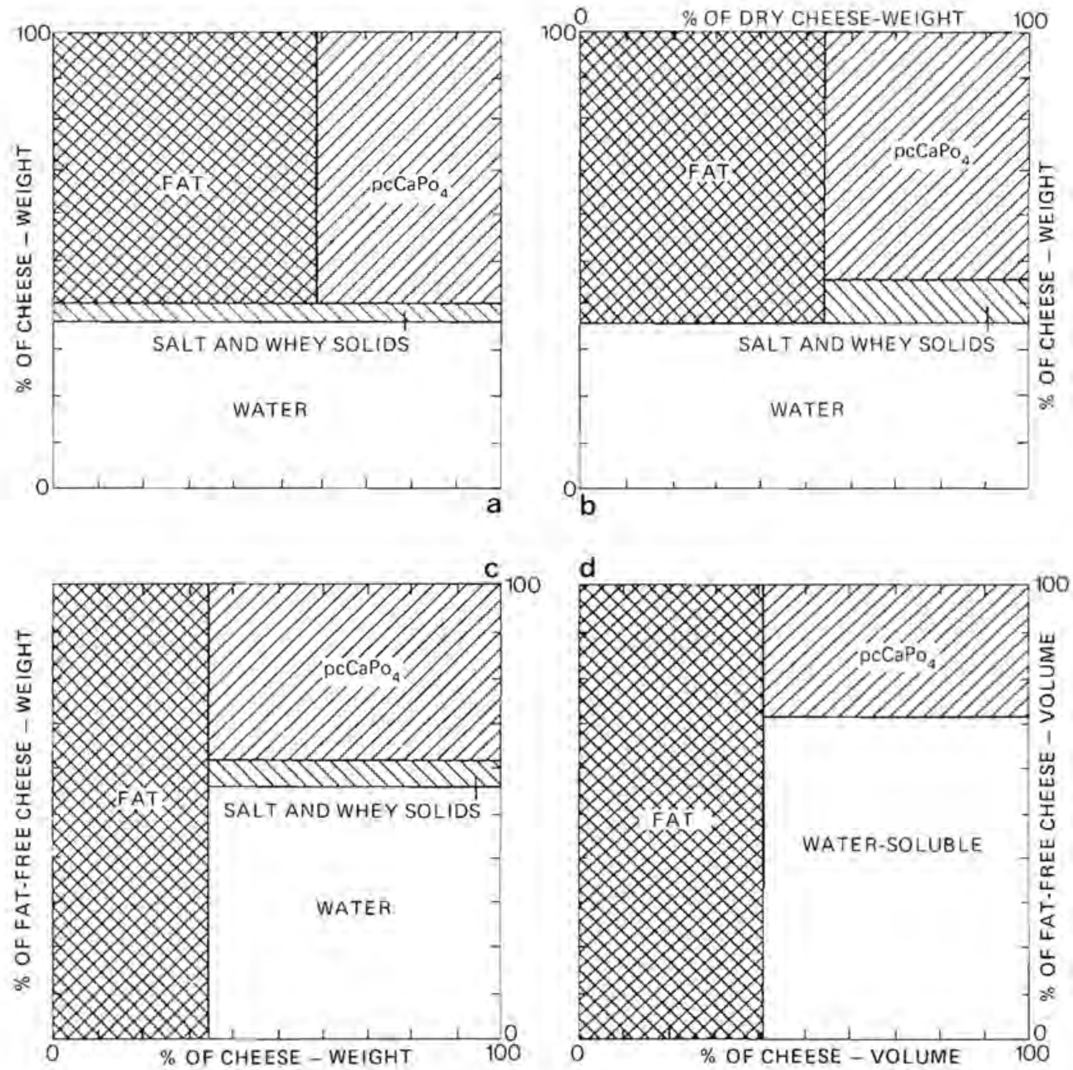

Figure 2. Representations of Cheddar cheese: A, where whey solids, salt, and moisture are distributed proportionally to fat and casein, B, where whey solids and salt are included only with casein and where moisture is distributed proportionally to fat and to fat-free cheese; C, where moisture, salt, and whey solids are included only with casein; and D, as in C, but represented on a volume basis. Composition of cheese: 37% moisture, 33.9% fat, 24.9% pcCaPO$_4$ and 4.2% salt and whey solids (wt, wt).

TABLE 1. General predictive formulas for yield of cheese.

Type A. Formulas where salt, whey solids, and moisture are distributed proportionally to both fat and para-casein.

Descriptive Formula [3]
$$Y = \left[\frac{\text{Fat} + \text{pcCaH}_2\text{PO}_4 \text{ in cheese}}{(\text{Fraction of fat and pcCaH}_2\text{PO}_4 \text{ in dry cheese})}\right]\left[\frac{1}{1 - M}\right]$$

A(a) [4]
$$Y = \left[\frac{F \cdot K_f + C \cdot K_c}{1 - SDC - \left[\frac{Msef}{1 - M}\right]\left[\frac{WS}{1 - WS}\right]}\right]\left[\frac{1}{1 - M}\right]$$

A(b) [5]
$$Y = \frac{F \cdot K_f + C \cdot K_c}{1 - SC - M - [(Msef \cdot WS / (1 - WS)]}$$

Type B. Formulas where salt and whey solids are included only with para-casein and moisture is distributed proportionally to fat and fat-free dry cheese.

Descriptive Formula [6]
$$Y = \left[\text{Fat in cheese} + \frac{\text{pcCaH}_2\text{PO}_4 \text{ in cheese}}{(\text{Fraction of pcCaH}_2\text{PO}_4 \text{ in fat-free dry cheese})}\right]\left[\frac{1}{1 - M}\right]$$

B(a) [7]
$$Y = \left[F \cdot K_f + \frac{C \cdot K_c}{1 - \frac{SC + \left[\frac{Msef \cdot WS}{1 - WS}\right]}{1 - FC - M}}\right]\left[\frac{1}{1 - M}\right]$$

B(b) [8][1]
$$Y = \left[F \cdot K_f + \frac{C \cdot K_c}{\frac{1 - SDC - FDC - \left[\frac{WS}{1 - WS}\right]\left[\frac{Msef}{1 - M}\right]}{1 - FDC}}\right]\left[\frac{1}{1 - M}\right]$$

Type C. Formulas where salt, whey solids and moisture are combined only with para-casein, with moisture as moisture in fat-free cheese (MFFC).

Descriptive Formula [9]
$$Y = \left[\text{Fat in cheese} + \frac{\text{pcCaH}_2\text{PO}_4 \text{ in cheese}}{(\text{Fraction of pcCaH}_2\text{PO}_4 \text{ in fat-free dry cheese})}\right]\left[\frac{1}{1 - MFFC}\right]$$

C(a) [10]
$$Y = F \cdot K_f + \left[\frac{C \cdot K_c}{1 - SFFDC - \left[\frac{MFFCsef}{1 - MFFC}\right]\left[\frac{WS}{1 - WS}\right]}\right]\left[\frac{1}{1 - MFFC}\right]$$

C(b) [11]
$$Y = F \cdot K_f + \frac{C \cdot K_c}{1 - SFFC - MFFC - (MFFCsef \cdot WS) / 1 - WS)]}$$

[1]Similar to formula of Posthumus et al. (33).

TABLE 2. Other predictive formulas for yield of cheese including those derived from actual yields in cheese making.

Type A formulas

[12] Van Slyke and Price (51) $$Y\ (\text{Cheddar}) = \frac{(.93F + C - .1)\ 1.09}{1 - M}$$

[13] Lelièvre et al. (29) $$Y = \frac{F \cdot K_f + .94\ (.97C) + .78}{1 - M}$$

Type B formulas

[14] Posthumus et al. (42) $$Y\ (\text{Gouda}) = \left[F \cdot K_f + \frac{\dfrac{(\alpha \cdot P - .022)}{1 - SDC - FDC - \left[\dfrac{WS}{1 - WS}\right]\left[\dfrac{M}{1 - M}\right]}}{1 - FDC} \right] \left[\frac{1}{1 - M}\right]$$

[15] Lolkema (30) $$Y\ (\text{Gouda}) = [F \cdot K_f + 1.2142P - 1.149\ K_w \cdot P_w] \left[\frac{1}{1 - M}\right]$$

Type C formulas

[16] Modified Van Slyke and Price (Appendix 7) $$Y\ (\text{Cheddar}) = .93F + \left[\frac{1.1682C}{1 - MFFC}\right]$$

Type E Formulas derived from actual yields in cheese making

[17]	Babcock (3)	$Y = 1.1F + 2.5C$
[18]	McDowall (37) (Based on Walker casein)	$Y = 1.189F + 2.084C$
[19]	McDowall (37) (Based on AOAC casein)	$Y = .98F + 2.42C$
[20]	Modified McDowall (10)[1]	$Y = 1.21F + 2.109C$[1]
[21]	Eino (14)	$Y = 1.135F + 2.111C - .171$
[22]	Banks et al. (5)	$Y = 1.32F + 1.32C + 1.58$

Other formulas

[23] Maubois and Mocquot (35, 36, 47) $$Y_{max}\ \text{unsalted} = \frac{(\mu \cdot C10)\ 10\ (200 - 300\ WS)}{200\ (100DC - 100FC) - 200WS\ (100 - 100FC)}$$

[1]Based on $Y = 1.21F + 1.62P$, $\lambda = .768$.

from the literature: VSP (51, 52) and a modified version of VSP (15); Lelièvre et al. (29); Banks et al. (5); PBK (42); Lolkema (30); Babcock (3); McDowall (37); Eino (14); and Maubois and Mocquot (35, 36, 47). Table 3 lists the equations of types A, B, C, and E from Tables 1 and 2, which have been reduced to simple equations using the constants in Appendix 2.

Type A Formulas

In a type A formula (Table 1, Formula [3]; Figure 2a), salt, whey solids, and moisture are distributed proportionally to both fat and para-casein. An important feature is that, as a predictive formula, the moisture (M) content of the cheese is indicated, as compared with indication of moisture in fat-free cheese (MFFC) in type C formulas.

Type B Formulas

In a type B formula (Table 1, Formula [6]; Figure 2b), salt and whey solids are distributed only to the casein; moisture is portioned to both fat and para-casein. The type B formula considers para-casein, calcium phosphate, salt, and whey solids together; these can then be compared to fat as in fat in dry cheese (fat in DM). This conforms to many cheese standards. It is apparent that any of the fat-free dry components (para-casein, CaH_2PO_4, salt, and whey solids) could substitute for each other in the type B concept of cheese and cheese yield.

Like type A formulas, moisture content is indicated. Other factors are also indicated, but in different ways, such as fat in cheese (FC) and salt in cheese (SC) in type B(a) and salt in dry cheese (SDC) and fat in dry cheese (FDC) in type B(b).

The type B(a) formula uses a constant FC. The type B(b) formula is similar to that of PBK; it uses a constant FDC. In milk of variable fat and casein content, the resulting cheese would not have a fixed FDC or a fixed FC. The FDC and FC can, however, be estimated in a predictive formula by a series of iterative cyclical calculations (see Appendix 3); this exercise is relatively simple using a computer. In practice, a constant FDC or FC can be used where milk is standardized to a constant ratio of casein : fat and moisture is constant.

Type C Formulas

In type C formulas (Table 1, Formula [9], Figure 2c), salt, whey solids, and moisture are portioned only to para-casein and, as a result, moisture appears in the equation as MFFC. The MFFC instead of M is the important feature of this type of formula.

Type D Formula

Type D formulas were not developed. They would be based on volume. They would seem to be theoretically more appropriate, because texture, etc. depends on spatial relationships, which in turn depend on volume. The water-soluble fraction in cheese should be considered instead of water alone. The volume occupied by fat is greater than that expressed by weight. The volume occupied by paracasein, per se, is less, although this is complicated by the para-casein occupying a greater volume by inclusion of water and water-solubles within its micelles. Type D formulas are not pursued at this time.

The Factor K_c

Some formulas in Table 1 use K_c, which represents a proportionality factor for the fraction of milk casein retained in the curd. It depends also on losses of para-casein in curd fines and in whey; retention of CaH_2PO_4 in the complex of para-casein and calcium phosphate; and losses of the glycomacropeptide (GMP) through the action of chymosin.

The PBK paper highlighted the importance of the loss of GMP and of the retention of Ca and P and curd losses in retention of casein in cheese. Van Slyke (48) recognized the solubilization of some casein and curd losses in cheese making, which is part of the .1 in the VSP formula.

The K_c was estimated for Cheddar and Gouda cheeses as 1.01908 and 1.03038 where the solute-exclusion factor (sef) (see later) is 0 and as 1.01813 and 1.02956 where sef is .5 (Appendices 2 and 4).

Type E Formulas

Tables 2 and 3 list six type E formulas that have been developed from cheese making practices designed to produce cheese of uniform quality by procedures as constant as possible. Four formulas are similar (Table 3): Constants for fat in the Babcock (3), McDowall (Walker) and (AOAC) (37) and Eino (14)

TABLE 3. Summary of formulas for Cheddar and Gouda cheeses reduced from those in Tables 1 and 2 to composition of milk[1] and cheese in Appendix 2.

Formula	Reference	Formula	Yield[1]	Ratio of factors for C and F
Type A formula				
Cheddar cheese				
[24]	Van Slyke and Price (51)	Y = 1.609F + 1.661C	9.885	1.03
[25]	General A(a) and A(b): sef[2] = 0	Y^3 = 1.584F + 1.735C	9.977	1.10
[26]	General A(a) and A(b): sef = .5	Y^4 = 1.562F + 1.710C	9.834	1.09
[27]	Lelièvre et al. (29) (.37 *M*)	Y = 1.476F + 1.447C + 1.238	10.117	.98
Gouda cheese				
[28]	General A(a) and A(b)	Y^3 = 1.718F + 1.891C	10.843	1.10
[29]	General A(a) and A(b)	Y^4 = 1.702F + 1.871C	10.737	1.10
Type B formulas				
Cheddar cheese				
[30]	Posthumus et al. (Appendix 6)	Y = 1.476F + 1.886C	9.961	1.28
[31]	Lolkema (Appendix 7)	Y = 1.476F + 1.986C	9.985	1.28
[32]	General B(a) and B(b)	Y^3 = 1.476F + 1.892C	9.997	1.28
[33]	General B(a) and B(b)	Y^4 = 1.476F + 1.834C	9.834	1.24
Gouda cheese				
[34]	Posthumus et al. (42)	Y = 1.596F + 2.031C	10.807	1.27
[35]	Lolkema (30)	Y = 1.596F + 2.040C	10.831	1.28
[36]	General B(a) and B(b)	Y^3 = 1.596F + 2.044C	10.842	1.28
[37]	General B(a) and B(b)	Y^4 = 1.596F + 2.005C	10.736	1.26
Type C formulas				
Cheddar cheese				
[38]	Modified Van Slyke and Price (15)	Y = .93F + 2.652C	9.883	2.85
[39]	General C(a) and C(b)	Y^3 = .93F + 2.690C	9.977	2.89
[40]	General C(a) and C(b)	Y^4 = .93F + 2.632C	9.832	2.83
Gouda cheese				
[41]	General C(a) and C(b)	Y^3 = .9362F + 2.875C	10.838	3.07
[42]	General C(a) and C(b)	Y^4 = .9362F + 2.836C	10.735	3.03
Type E formulas observed in cheese making				
Cheddar cheese				
[17]	Babcock (3)	Y = 1.1F + 2.5C	10.120	2.27
	McDowall (37)			
[43]	based on Walker casein (3423 vats) (.37 M)[5] from Formula [19]	Y = 1.233F + 2.161C	9.764	1.75
[44]	based on AOAC casein (687 vats) (.37 M)[5] from Formula [20]	Y = 1.016F + 2.510C	9.842	2.47
[45]	Eino (14) (.37 M)[6] Formula [22]	Y = 1.172F + 2.180C − .177	9.414	1.86
[46]	Banks et al. (5) (.37 M)[7]	Y = 1.367F + 1.367C + 2.636	9.925	1.00

[1]Milk composition for Cheddar was 3.6% fat, 2.464 casein, and for Gouda was 3.37% fat, 2.6727 casein.

[2]sef = Solute-exclusion factor.

[3]sef = 0.

[4]sef = .5. The actual simpler formula is more complicated as in the example in Appendix 3, Formula [53]. The formulas are further simplified assuming that, in the denominator, C = 2.464 and Y is the estimated yield.

[5]Formula was adjusted for whey from .349 to .37 M as in footnote 7.

[6]Formula was adjusted for whey from .3514 to .37 M as in footnote 7.

[7]Formula was for whey from .35 to .37 M and .065 whey solids: $Y_{adj} = Y_{obs}(1 - .35 - .065)/(1 - .37 - .065)$ (34) (Formula [47]).

formulas were 1.1, 1.23, 1.02, and 1.17 and constants for casein were, 2.5, 2.16, 2.51, and 2.18, respectively. These factors were closer to those of type C formulas than to those of types A and B. Factors for fat in the types A, B, and C general formulas for Cheddar cheese (Table 3, sef = .5) were 1.56, 1.48, and .93 for fat, and for casein, 1.71, 1.83, and 2.63, respectively.

Early (55) and more recent (20, 28) work showed that extra fat in cheese seemed to carry with it only small, although significant, amounts of moisture (10 to 20% of the weight of fat). Added to the recovery of fat of .93 (plus .1 to .2), this corresponds closely to the fat constants in the type E formulas. Nevertheless, the most important factor in moisure retention in cheese and in yield is casein, and it is responsible for retaining more other components in fat-free cheese than its own weight (19).

Other formulas have been developed but have not been considered here because the forms of the equations made them difficult to compare. The Maubois and Mocquot formula 35, 36) is based on the sponge theory (47) that cheese consists of a para-casein matrix that acts as a sponge for the other components. Banks et al. (4, 5) developed several formulas with constants independent of fat and casein and similar to those developed by Eino (14); one of these, Formula [22], gave the best fit for yields from both standardized and unstandardized milk; in the mathematical development of the formulas, fat and casein were given equal weighting. Lelièvre et al. (27, 29) developed Formula [13] by blending formulas by Van Dam and Janse and by Van Slyke and Price [51].

Comparison of the Formulas and Discussion

Whey Solids and Salt in Formulas. The general formulas (types A, B, and C) and the PBK formula have a factor for whey solids (Tables 1 and 2). This is significant where moisture varies considerably, as pointed out by Maubois and Mocquot (34). Table 4 shows that the level of whey solids in cheese increases as moisture increases. Not only do levels of fat, $pcCaH_2PO_4$, salt, and whey solids change when moisture changes, but their relative proportions also change.

TABLE 4. Amount of whey solids and whey in cheese of increasing moisture content, and yield[1] of cheese with the Van Slyke and Price (VSP) formula and with the type A(b) formula [5] when salt is included in different forms.

Cheese composition				VSP Formula [12]			Percentage higher increase in yield than with the VSP formula using the type A(b) formula based on		
Moisture	Solute-excluding moisture[2]	Whey solids[3,4]	Whey[5]	Yield[6]	Yield increases when M increases by .01 to level indicated		SDC[7] (.027)	SC[8] (.017)	SC:M[9] = Constant (.0459)
				(kg)			(%)		
.32	.1304	.0132	.2032	9.156	.133	1.47	.160[5]	.199	.264
.37	.1195	.0174	.2683	9.883	.154	1.59	.188	.234	.312
.47	.0977	.0259	.3984	11.747	.218	1.89	.272	.340	.458
.57	.0760	.0344	.5286	14.479	.329	2.33	.427	.539	.737

[1]Milk of average composition (Appendix 2), M = .37.

[2]sef = .5; solute-excluding moisture = sef ($\mu \cdot$ C – peCL)/Y; use SC in calculating Y.

[3]Msef · WS(1 – WS).

[4]A .01 increase in moisture plus whey solids.

[5]Moisture minus solute-excluding moisture plus whey solids.

[6]For example, the increase in yield from .31 to .32 was .14725 kg (type A(b) formula); (.14725 – .1327) (100/9.156) = .160%.

[7]Salt in dry cheese.

[8]Salt in cheese.

[9]Salt : moisture in cheese.

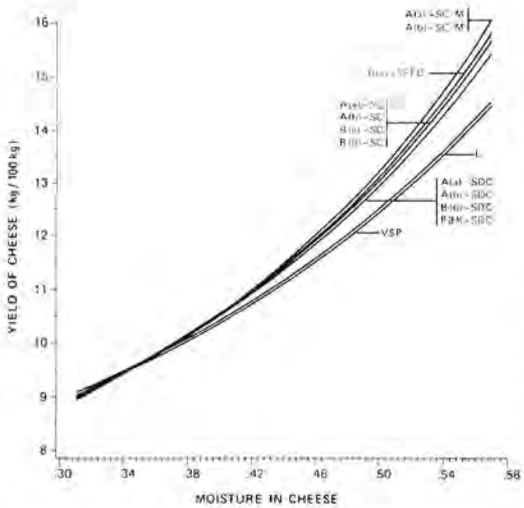

Figure 3. Yield of cheese when moisture is varied, showing the effect of a separate factor for whey solids and of different forms for factors for salt. The formulas are: Van Slyke and Price (VSP); Lolkema (L) (30); Posthumus et al. (PBK); types A(a), A(b), and B(b) using alternative forms of salt fraction in cheese (SC) and in dry cheese (SDC); type B(a) using SC and salt fraction in fat-free cheese (SFFC); types A(a) and A(b) in which ratio of salt to moisture constant of .04595 (.017/.37), i.e., SC = .04595M.

Figure 3 shows estimated yields given by 13 formulas for cheese of moisture contents from .31 to .57 M using formulas where sef = 0. Most include whey solids as a separate factor; those of VSP and Lolkema (30) do not. Different forms of salt are used in these formulas: SC, SDC, salt in fat-free cheese (SFFC), and SC:M (the last as a constant) (See Appendix 1).

Those formulas that have a separate factor for whey solids result in higher increases in yield than the VSP (51) and Lolkema (30) formulas that do not. It should be recognized, however, that the Lolkema formula (30) has a factor for lactose that should change for different varieties of cheese.

With respect to the form of salt in the formula, the two in which SC:M is constant result in the largest and identical increases, followed in decreasing order by SFFC, SC, and SDC. With SC:M, both salt and whey solids increase in direct proportion to moisture. With SC, the salt is constant only in the cheese; the added water dilutes the dry cheese. With SDC, the salt is constant only in the dry cheese, which decreases as the moisture increases. The SC is likely to be the form of choice for salt in the formulas because standards usually specify salt in cheese. However SC:M may also be useful where the standard is for SC:M (19, 20).

Figure 3 gives trends. Table 4 gives comparable data for the increases in yield for the VSP and type A(b) (sef = .5) formulas where moisture at various concentrations is increased by 1% and where salt is in the form of SDC, SC, and SC:M = constant. There is no separate factor for whey solids in the VSP formula; whey solids are in the type A(b) formula. An increase from .36 to .37 M results in a yield increase of .154 kg (1.59%) with the VSP formulas. The type A(b) formula results in a further increase, ranging from .188 to .312%, depending on the form of salt in the formula. The yield increases are larger in all categories as the level of moisture in the cheese increases. For example, with SC, yield increases with the A(b) formula by 1.67 (1.47 + .199), 1.82, 2.23, and 2.87% when moisture increases .01 (1%) to .32, .37, .47, and .57. Figure 3 also illustrates the importance of moisture in cheese in determining yield.

Obviously, whey solids in moisture are important in yield. Maubois and Mocquot (34) recommended adjustment (adj) of yield for whey rather than for moisture alone:

$$Y_{adj} = \frac{Y_{obs}\,(DC_{obs} - WS)}{(DC_{adj} - WS)} \qquad [47]$$

where obs = observed.

In summary, adjustment of yields to constant composition should include not only moisture but also whey solids and perhaps salt. Careful consideration should be given to whether constant M or MFFC is used and to the form of salt in the formulas.

Solute-Exclusion Moisture. The level of whey solids in cheese is not certain. Nor is it certain that all whey components are retained in cheese in the same proportion. There is evidence that all water associated with para-casein is not free to act as a solvent (Appendix 9). Para-casein has 2.5 times its weight of water associated with it in such a way that the water is unavailable as a solvent for whey proteins (23). Similarly, para-casein has .5 times its weight of water associated with it in such a way that it does not act as a solvent for lactose (38, 39, 53, 54). In this paper, sef of .5 is used for all whey components. The general formulas are given with and without sef. Appendix 9 gives further details.

Calcium Phosphate. In addition to whey solids, calcium phosphate (in association with para-casein) forms a significant part of cheese (42). The CaH_2PO_4 in the para-casein complex is estimated in Cheddar cheese (.37 M) to be 1.61% (Appendix 4) and in Gouda cheese to be 1.86%. Calcium and PO_4 dissociate from the casein micelle as the pH is lowered. Therefore, cheese varieties made at higher pH would be expected to have higher CaH_2PO_4. This suggests that increased yield due to the retention of CaH_2PO_4 in cheese may result from addition of $CaCl_2$ (other Ca salts that do not decrease pH might be better); addition of phosphate salts; or use of starter concentrates that result in coagulation of milk at a higher pH. Posthumus et al. (42) were the first to emphasize the importance of Ca and PO_4 in the yield of cheese. Others have studied other aspects of their importance: Czulak et al. (11) studied retention of Ca in cheese and Lawrence et al. (26) studied Ca concentrations and cheese properties. It is evident, of course, that Ca and PO_4 are associated with the para-casein during coagulation and the early stages of cheese making; in the final cheese at a lower pH, at least some is dissociated into the water phase (11).

Types A, B, and C Formulas. Figure 4 shows only a slight difference between formula types A and B in their predicted yields from milks of different composition, whereas those of type C are quite different, which is expected because type C formulas place different emphasis on fat and casein in the milk.

When only fat is increased (Figure 4a) and casein is constant, yield increases are greater in the order of C (lowest), then B, and then A. The difference between A and C for an increase of .1% fat is .66% yield.

When only casein is increased (Figure 4b) and fat is constant, yield increases are greater in the reverse order of A (lowest), B, and C. The difference between A and C for an increase of .1% casein is .99% yield.

When milk casein and fat both increase in a ratio of casein : fat of .4 (50) (Figure 4c), yield increases are greater with C lowest, B intermediate, and A highest. The difference between A and C for an increase of .4% fat and .16% casein is 1.09%. The differential per unit change in a component is less than in Figures 4a and 4b because the other component changes also.

When fat and casein both increase in a constant casein : fat ratio of .68444, as when milk is standardized, there are no differences among the formulas (Figure 4d). If the compositional factors are approximately constant, then types A, B, and C formulas give similar yields. The ratio of .68444 is that of milk containing 3.6% fat and 2.464% casein (Appendix 2). However, if the milk is standardized to casein : fat ratio other than .68444, the ratio at which types A, B, and C formulas are equivalent, then the formulas result in different differentials (not shown).

The type B formula would be preferable to type A, where cheese of a constant FDC is made. It conforms to Model B (Figure 2b) where the moisture is distributed proportionally to the fat and a combination of para-casein complex, whey solids, and salt. The type B formulas result in a slightly greater emphasis on casein with a higher casein : fat ratio (Table 3).

There is little difference among the type B formulas. The Lolkema type formula has application where protein in milk and whey can be measured side by side. The PBK and general formulas would apply but require determination of casein in milk either directly or from seasonally derived α (conversion factor for milk protein to the complex of para-casein and CaH_2PO_4) or λ (casein numbers) of local milk protein (42). The estimation of casein from protein is important and will be discussed in a later section.

Comparing the type C formula with types A and B formulas is more difficult. Types A and B apply where cheesemaking conforms to making cheese of constant moisture. The type C formula applies where a constant MFFC is needed. These applications will be discussed later in this paper.

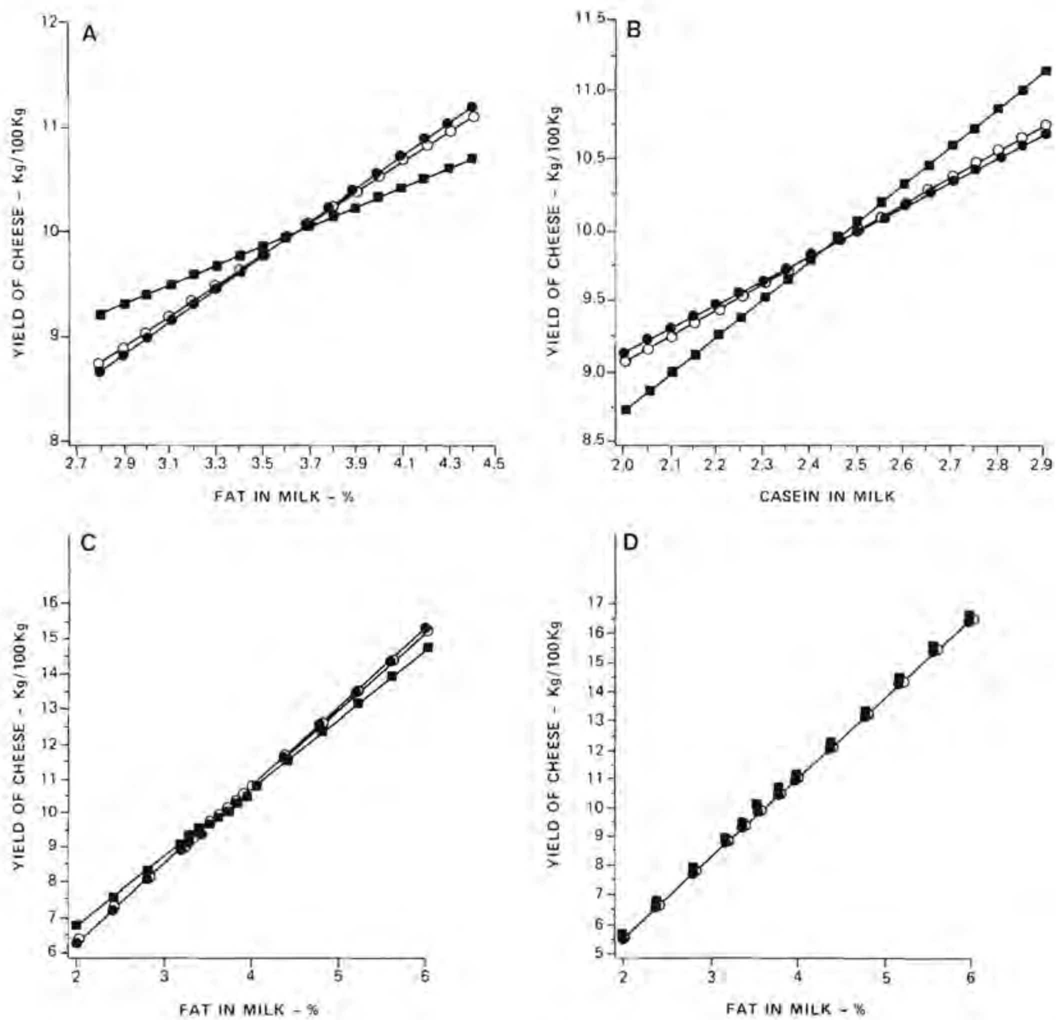

Figure 4. Comparison of general predictive formulas (sef = 0) of types A (closed circles), B (open circles), and C (closed squares) on yields of cheese from milk: A, varied fat, constant casein; B, varied casein, constant fat; C, varied fat and casein in ratio of 1 : .4; D, varied fat and casein at constant casein : fat ratio of .68444.

Certain assumptions were made in deriving the formulas in Table 1. There is a need, therefore, that such formulas be verified by experimentation using current methods of analysis and measurement. For example, we understand that the VSP formula (52) was derived from data where protein was estimated from nitrogen by a factor of 6.25 (49), instead of 6.38; thus, the VSP formula would be expected to overestimate yield by 2.1% today. Further, our best estimations of The Netherlands formulas for Cheddar cheese give slightly higher yields than the general formulas (sef = .5) (Table 3).

Casein to Fat Ratio in Formulas. Table 3 lists the various equations reduced to only factors for F and C, using the constants in Appendix 2. As expected, the relative contributions to yield for casein and fat differ and are about 1.09 : 1, 1.24 : 1, and 2.83 : 1 for types A, B, and C formulas (sef = .5). The relative contributions in type E formulas [17], [44], and [45] are 2.27: 1, 2.47 : 1, and 1.86 : 1, where casein is measured directly. These are closer to type C formulas. The relative contribution where casein is estimated by the Walker formol titration is 1.75, which is closer to type B formulas.

Estimated Yields. Gouda cheese yields ranged from 10.735 to 10.843 kg/100 kg of standardized milk. The two formulas for Gouda cheese, those of Lolkema (30) and PBK, are based on commercial data and give close yields, 10.831 and 10.807 kg, from the same milk and other conditions.

For Cheddar cheese, yields ranged from 10.120 to 9.414 kg of milk. Where possible, the formulas were adjusted to give cheese with moisture of .37. The general (sef = .5 and 0), Banks et al. (5), Lolkema (30), and PBK formulas ranged from 9.823 to 9.985 kg. The VSP formula predicted a yield of 9.885 kg, which was .9% lower than that from the general formula (sef = 0) and .5% higher than that of the general formula (sef = .5). As noted (Figure 3, Table 4), most would be close to those of the general formulas only at .37 M.

Theoretical Versus Practically Derived Formulas. There is merit in using a theoretically derived formula for predicting yield as compared with one derived from actual cheese making. There is a danger of building into the latter errors inherent in analysis and cheese making at the time of the experiment. Accuracy demands that the scope of the experiment covers all essential situations; for example, Banks et al. (5) developed different formulas for seasonal and standardized milk. Such formulas may predict too low or too high. The theoretical formula can serve as a target. However, an accurate theoretical formula depends on full knowledge of all factors in transfer of milk constituents to cheese, which may not yet be the case (Appendix 9).

An important application is in comparing an actual yield to a theoretical yield of cheese of the same composition; this could be used, presumably, instead of adjusting the actual yield to that of cheese of a target or standard composition. In this same way, theoretical yields of two processes could be compared in which, for example, different amounts of whey proteins or of CaH_2PO_4 are retained or in which rennet is not used (Modler and Emmons, unpublished data).

Moisture in Fat-Free Cheese

The MFFC is important for cheese quality and for use as a regulatory tool. The quality of cheese is related more closely to MFFC than to moisture alone (20, 25, 37, 41, 43, 56); thus it is logical that MFFC would be more important than moisture alone if salt in moisture is also considered and if the casein and fat vary relative to each other in the cheese.

Another aspect of quality is firmness. The MFFC is the major factor for classifying cheese varieties according to firmness by the International Dairy Federation (7) (Table 5) and was developed as a result of early work by Schulz et al. (44) and Kay (24). This classification is based on the fact that cheeses of different fat content would have approximately the same firmness if they had the same MFFC, even though there is some evidence that reduced fat cheeses are firmer at the same MFFC (16).

TABLE 5. Descriptive nomenclature of cheese for firmness based on moisture in the fat-free cheese (MFFC) (7).

Description	MFFC
	(%)
Soft	>66
Semi-soft	61–68
Firm	49–63
Hard	<51

The MFFC concept as a regulatory tool has been recognized in some countries. For example, the Federal Republic of Germany uses it in defining moisture content of Camembert cheese of different fat contents (FDC) (Table 6) (33). The MFFC is 70% in the three cheeses containing 30, 40, and 45% FDC; equivalent moisture and fat contents are 62, 58, and 56% and 11, 17, and 20%. It would be inappropriate to regulate the same moisture content for fat-reduced cheese as for the full-fat version of the variety.

The type E formulas, derived from actual cheese making, are closer to type C formulas using MFFC than to type A and type B

TABLE 6. Composition of Camembert cheese containing different levels of fat in the Federal Republic of Germany (33).

Regulations		Calculations		
FDC[1]	DM	Moisture	Fat	MFFC[2]
(%)				
30	38	62	11.4	70.0
40	42	58	16.8	69.7
45	44	56	19.8	69.8

[1]Fat in dry cheese.

[2]Moisture in fat-free cheese.

formulas using M (Table 3). Recent data by Amantea et al. (1) from 36 vats of commercial cheese showed less variation in MFFC than in M as illustrated in CV. Mean values, SD, and CV for MFFC were 55.01, ± .722, and ± 1.31% and for M were 36.59, ± .897, and ± 2.45%. Calculations from the data of Eino (14) showed means, SD, and CV of 50.96, ± .99, and ± 1.94% for MFFC and 35.14, ± 2.03, and ± 5.78% for M. Because the CV of M was considerably higher than the CV of MFFC in both studies, one concludes from these and other data (20) that cheese making practices give more uniform MFFC than M from milk of varied composition.

For technological reasons and for quality, there is considerable merit in cheese standards for both FDC and MFFC instead of fat and moisture. The FDC is retained in standards for cheese varieties in many countries. The FDC is easily and accurately predicted by standardizing milk to a constant casein : fat ratio. If this is done, then it is apparently easier to control moisture or MFFC and quality. If milk is not standardized, standards for MFFC are more appropriate, as noted earlier. The MFFC has the further advantage of better relating cheese varieties with different FDC.

Relation Between Protein and Casein

Most of the preceding formulas depend on analysis of milk for casein. As yet, there is not a simple, accurate procedure that has been applied for casein in milk. Various simple procedures have been developed such as the formol titration (37). More recent procedures depend on analysis of milk before and after removal of casein (6, 8, 23, 46); they have promise as practical tests.

Protein has been used in predictive yield formulas such as those of McDowall (37) and PBK. The problem is that casein does not form a constant proportion of protein in milk. Instead, it varies seasonally about .02 from .757 to .778 (2). Variations can occur among factories; a Dutch report showed consistent differences in annual means of casein numbers from five factories, from .759 to .778 (2); McDowall (37) observed a range from .770 to .775 (mean of .772) among seven factories determined by formol titration; he also observed a range from .733 to .784 (mean of .766) among the same factories using AO AC Kjeldahl determinations. Cerbulis and Farrell (9) showed differences among breeds, .803 for Jerseys and .784 for Holsteins.

The question of accuracy of determination of the casein numbers is a real one in considering published values. Apparently the casein number can also decrease with storage time before analysis due to bacterial and native enzymes (13). Nevertheless, PBK appear to have successfully used, in commercial practice, seasonally varying casein numbers in their predictive yield formula.

Where applicable, the procedure of Lolkema (30) obviates the variations in casein numbers by estimating casein from protein measurements on milk and whey. The following section considers this for the general formulas.

Application of Analyses of Protein in Milk and Whey to Yield Formulas

Lolkema (30) introduced the measurement of protein in milk and whey to yield formulas. This measurement can be important in certain applications, particularly in cheese factories where both analyses can be easily performed. He used it to estimate recovery of total protein in milk as cheese, including both para-casein and whey proteins. Recovery was $(P - K_w \cdot P_w)$ where K_w was the proportion of whey to milk during cheese making (Appendix 6).

It seems possible to use such analyses to estimate casein content of milk. A possibility is to assume that the protein content of whey is uniform in all parts of the coagulated milk which are not fat or para-casein. Where λ = proportion of casein in milk protein (e.g., .77) and μ = proportion of para-casein in casein [e.g., .96), then:

fraction of para-casein in milk = $\lambda \cdot \mu \cdot P$. [48]

Where K_m = fraction of whey in milk

= fraction of fat-free, para-casein-free portion in milk

$$= 1 - \frac{\lambda \cdot \mu \cdot P}{100} - \frac{F}{100}, \qquad [49]$$

then para-casein in milk (kg/100 kg)

$$= P - K_m \cdot P_w$$

$$= P - \left[1 - \frac{F + \lambda \cdot \mu \cdot P}{100}\right] P_w \qquad [50]$$

and C = Formula [50]/μ. [51]

Formula [51] might then be substituted for C in any of the formulas of Tables 1, 2, and 3. The K_w differs from K_m in that K_w is the proportion of cheese whey to milk and K_m is the proportion of cheese whey in milk. Karman et al. (23) advocate the use of large exclusion factors for whey nitrogen from bound water in para-casein.

Are Yield Formula Necessary?

Others have examined the question of whether yield formulas are necessary and concluded that they are not useful. For example, some prefer to judge the performance of cheese factories on the basis of conformity to a constant (or range of) MFFC (26) and of monitoring fat and casein losses in whey (20, 40); inaccuracies in measuring fat and casein in milk result in inaccuracies in yield estimations (20). However, Lolkema (30) and PBK (42) indicate that the use of predictive formulas is a useful commercial practice in monitoring efficiency (yield and losses) during and after cheese making. Lolkema (30) used his formula for predicting moisture content of cheese from the observed yield; as such, values from average cheese makaing are needed for constants such as K_f and pcCL; average values are not necessarily desirable for target values in assessing efficiency or in trying to improve it.

Observed Yield as Percentage of Theoretical Yield

Yield formulas, properly applied, may have a place as targets for cheesemakers to measure performance as a percentage of theoretical yield. Such a target may reveal hidden losses during manufacture and improve overall efficiency of cheese production.

In research, it might be useful to express yield and relative yield as percentages of theoretical yield. It might obviate differences in composition of milk among trials and enable comparison of treatments applied to different milks, where it is impossible to make paired vats from the same milk.

Pricing Formulas for Milk

Yield formulas are necessary for multiple-component pricing in establishing the relation between milk composition, yield, and milk price (10). Pricing formulas for milk are ideally based on yield of products. Considering the different emphasis placed on fat and casein (protein) in the different formulas in Table 3, the selection of the correct formula would seem to be critical. This will be the subject of a subsequent paper.

Summary and Conclusions

Various yield formulas were described or developed. They have application for predicting yield of cheese of constant composition or for comparing actual and theoretical yields after analysis of cheese. They are developed with a model of cheese as three phases of fat, para-casein, and water solubles.

Types A and B formulas apply when cheese is made to a constant moisture content. Any of types A, B, and C formulas appear to apply if milk is standardized to a constant C : F ratio and if appropriate constants are used. Type B formulas are preferable where cheese is made to a constant FDC, conforming to Model B.

Type C formulas have particular application when comparing yields from milks intentionally varied to produce reduced-fat variants of cheese varieties. They also apply better than types A and B formulas for making cheese of uniform composition from milk of variable composition without standardizing the milk. There is justification technologically for standards for cheese based on MFFC and FDC instead of moisture and fat.

Type E formulas were those obtained when relating actual yield to naturally variable fat and casein in milk. They are closer to type C formulas than to type A and type B formulas.

The types A, B, and C general formulas that were developed would be applicable to most varieties of cheese. Those with SC would be more applicable when a constant salt is desired.

The types A, B, and C formulas account for whey solids in cheese and have an important bearing on predicted yield when moisture varies. The formulas were adapted to include an sef for that portion of moisture bound to para-casein that does not contain whey solids.

Two formulas for Gouda cheese [Lolkema (30) and PBK (42)] are based on extensive commercial data and give close yields, 10.831 and 10.807 kg, from the same milk and other conditions. There is a need for research to refine constants in the general formulas in this paper more accurately for different varieties of cheese, e.g., the retention of whey solids.

Cheese yield formulas appear to have application as control procedures in predicting yields and as comparisons in assisting cheese makers to obtain maximum yields. They are central to developing pricing formulas based on yield of products from composition of milk. They may be useful in expressing yield of cheese as percentage of theoretical yield.

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APPENDIX 1

TABLE A1. Glossary of abbreviations and terms.

α	=	(.)	Conversion factor of protein in milk to $pcCaH_2PO_4$ in cheese (42).
C	=	(kg)	Casein content of milk = kilograms of casein/100 kg milk.
CF	=	(kg)	Fat in cheese from 100 kg of milk.
CM	=	(kg)	Moisture in cheese from 100 kg of milk.
CS	=	(kg)	Salt in cheese from 100 kg of milk.
CWS	=	(kg)	Whey solids in cheese from 100 kg milk.
CaH_2PO_4	=	(.)	Inorganic calcium and PO_4 in cheese.
DC	=	(.)	Fraction of dry cheese = (1 − M).
F	=	(kg)	Fat content of milk = kg fat/100 kg milk.
FC	=	(.)	Fat fraction of cheese.
FDC	=	(.)	Fat fraction of dry cheese = FC/(1 − M).
FFC	=	(.)	Fraction of fat-free cheese = (1 − FC).
FFDC	=	(.)	Fraction of fat-free dry cheese = (1 − M − FC).
K_c	=	(.)	Conversion factor of casein in milk to $pcCaH_2PO_4$ in cheese.
K_f	=	(.)	Conversion factor for fat from milk to cheese.
K_{ffdc}	=	(.)	Conversion factor of casein in milk to FFDC in modified VSP formula (15).
K_m	=	(.)	Fraction of whey in milk = fat-free, para-casein-free milk.
K_p	=	(.)	Conversion factor for protein from milk to cheese.
K_w	=	(.)	Fraction of whey to milk = milk minus cheese (30).
λ	=	(.)	Fraction of casein in total protein in milk.
M	=	(.)	Moisture fraction in cheese.
MFFC	=	(.)	Moisture fraction in fat-free cheese = M/(1 − FC).
Msef	=	(.)	Fraction of moisture in cheese that can act as solvent for whey solids = M − sef($\mu \cdot$ C − pcCL)/Y.
MFFCsef	=	(.)	Fraction of moisture in cheese that can act as solvent for whey solids = MFFC − (sef($\mu \cdot$ C − pcCL)/Y)/(1 − FC).
P	=	(kg)	Protein content of milk = kilograms of protein/100 kg milk.
pc	=	(.)	Fraction of para-casein in cheese.
$pcCaH_2PO_4$	=	(kg)	Complex of para-casein and CaH_2PO_4 in cheese from 100 kg of milk ($C \cdot K_c$).
pcCL	=	(kg)	Para-casein in curd fines (kilograms/100 kg of cheese milk).
P_w	=	(kg)	Protein content of whey = kilograms of protein/100 kg whey.
R_{ffdc}	=	(.)	Conversin factor of protein in milk to fat-free dry cheese (30).
SC	=	(.)	Salt fraction in cheese.
SDC	=	(.)	Salt fraction in dry cheese = SC/(1 − M).
SFFC	=	(.)	Salt fraction in fat-free cheese = SC/(1 − FC).
SFFDC	=	(.)	Salt fraction in fat-free dry cheese = SC/(1 − FC − M).
SWSFDC	=	(.)	Salt fraction in whey-solids-free dry cheese = SDC/(1 − WSC).
sef	=	(.)	Fraction of protein equivalent to the water associated with it that cannot act as solvent for solutes = .5 for whey solids.
μ	=	(.)	Fraction of para-casein in casein (.96) (42).
υ	=	(.)	Ratio of $pcCaH_2PO_4$ to pc = (pc + CaH_2PO_4)/pc = 1 + $CaPO_4$/pc (42).
WFFC	=	(.)	Fraction of whey in fat-free cheese.
WS	=	(.)	Solids fraction in fat-free, casein-free whey.
WSC	=	(.)	Whey solids fraction in cheese = $M\left[\frac{WS}{(1-WS)}\right]$.
WSDC	=	(.)	Whey solids fraction in dry cheese = WSC/(1 − M).

WSFDC	= (.)	Fraction of whey-solids-free dry cheese = (1 – M – WSC).
WSFFC	= (.)	Whey solids fraction in fat-free cheese = WSC (1 – FC).
WSFFDC	= (.)	Whey solids fraction in fat-free dry cheese = WSC/(1 – FC – M).
Y	= (kg)	Yield of cheese per 100 kg milk or yield of fraction of cheese (e.g., YDC).

APPENDIX 2

TABLE A2. Composition of milk and cheese, and various factors used in equations for Cheddar and Gouda cheese. The fat and protein contents are those of "average" milk and Cheddar cheese or of standardized milk and Gouda cheese used by Lolkema (30) and Posthumus et al. (42).

			Factors used for	
			Cheddar cheese	Gouda cheese
Composition of milk				
	F	=	3.6 kg[1]	3.370 kg
	P	=	3.20 kg	3.431 kg
	λ	=	.77	.779
	C	=	2.464 kg	2.6727 kg
	α	=	.786	.804
Composition of whey				
	WS	=	.065 kg	.042 (second whey)
	P_w	=	...	.954 kg
	K_a	=	.90	.89
	pcCL	=	.022 kg[2]	.022 kg[2]

Composition of cheese

		Van Slyke and Price formula	General[3] and other formulas		General and other formulas	
			sef = 0	sef = .5	sef = 0	sef = .5
Y	=	9.885 kg	9.977	9.837	10.850	10.737
M	=	.37	.37	.37	.4134	.4134
MFFC	=	.5595	.5569	.5609	.5829	.5854
FC[5]	=	.3387	.3356	.3404	.2908	.2938
FDC	=	.5376	.5327	.5403	.4957	.5009
FFDC	=	.2913	.2944	.2896	.2958	.2928
K_f	=	.93	.93	.93	.9362	.9362
K_c	=	...	1.01908	1.01813	1.03038	1.02956
SC	=	.0170	.017	.017	.0235	.0235
SDC	=	.0270	.0270	.0270	.0401	.0401
SFFC	=	.0257	.0256	.0258	.0331	.0333
SFFDC	=	.0584	.0577	.0587	.0794	.0803
WSC	=	.0257	.0257	.0176	.0185	.0130
WSDC	=	.0408	.0408	.0279	.0315	.0222
WSFFC	=	.0389	.0387	.0267	.0261	.0184
WSFFDC	=	.0882	.0873	.0608	.0625	.0444
CaH_2PO_4	=	.0149	.0161	.0161	.0186	.0186
pc[4]	=	.2355	.2356	.2389	.2352	.2377

[1]All kilograms are per 100 kg of milk or whey, except pcCL.

[2]Per 100 kg of milk.

[3]General formula [7], type B(a).

[4]By difference (1 – M – FC – SC – WSC – CaH_2PO_4).

[5]Calculated as $F \cdot K_f / Y$.

APPENDIX 3

General Formulas for Cheese

General formulas relating composition of milk and yield of cheese can be derived from the very general basic Formulas 1 and 2. This is basically that described in the graphs of Van Slyke and Price (51) and Van Slyke and Publow (52) and in Figure 2. The following are derivations of types A, B, and C (Table 1) that conform to the Models a, b, and c in Figure 2.

Type A Formulas

The descriptive type A formula, where whey solids, salt, and moisture are distributed proportionally to fat and para-casein, is:

$$Y = \left[\frac{\text{Fat in cheese} + \text{pcCaH}_2\text{PO}_4 \text{ in cheese}}{\text{Fraction of fat and pcCaH}_2\text{PO}_4 \text{ in dry cheese}}\right]\left[\frac{1}{1 - M}\right] \quad [3]$$

Type A(a) Formulas Based on 1/(1 – M)

$$Y = F \cdot K_f + C \cdot K_c + CS + CWS + CS$$

$$YDC = F \cdot K_f + C \cdot K_c + CS + CWS$$

Since CS = SDC · YDC and CSW = WSDC · YDC, then CS + CWS = (SDC + WSDC) YDC, and YDC (1 – SDC – WSDC) = F · K_f + C · K_c

$$YDC = \frac{F \cdot K_f + C \cdot K_c}{1 - SDC - WSDC}.$$

$$\text{Since } Y = YDC\left[\frac{1}{1 - M}\right],$$

$$\text{then } Y = \left[\frac{F \cdot K_f + C \cdot K_c}{1 - SDC - WSDC}\right]\left[\frac{1}{1 - M}\right].$$

$$\text{Since } WSDC = WSC\left[\frac{1}{1 - M}\right] \text{ and } WSC = \frac{M \cdot WS}{1 - WS},$$

$$\text{then } WSDC = \left[\frac{M}{1 - M}\right]\left[\frac{WS}{1 - WS}\right]$$

and

$$Y = \left[\frac{F \cdot K_f + C \cdot K_c}{1 - SDC - \left[\frac{M}{1 - M}\right]\left[\frac{WS}{1 - WS}\right]}\right]\left[\frac{1}{1 - M}\right]. \quad [4]$$

Type A(b) General Formulas

Because CS = SC · Y, CWS = WSC · Y, and CM = M · Y, then from Formula [2]:

$$Y = F \cdot K_f + C \cdot K_c + (SC + WSC + CM)\ Y,$$

$$Y = \frac{F \cdot K_f + C \cdot K_c}{1 - SC - WSC - M}\text{, and}$$

$$Y = \frac{F \cdot K_f + C \cdot K_c}{1 - SC - M - \frac{M \cdot WS}{1 - WS}}. \qquad [5]$$

Using factors from Appendix 2 and sef = .5, Formulas [4] and [5] can both be reduced to either

$$Y(\text{Cheddar}) = 1.561F + 1.710C \qquad [26]$$

$$\text{or } Y(\text{Gouda}) = 1.702F + 1.871C. \qquad [29]$$

Type B Formulas

The descriptive Type B formula, where whey solids and salt are included only with paracasein and where moisture is distributed proportionally to fat and to fat-free dry cheese, is:

$$Y = \left[F \cdot K_f + \frac{C \cdot K_c}{\text{fraction of pcCaH}_2\text{PO}_4 \text{ in FFDC}}\right]\left[\frac{1}{1 - M}\right] \qquad [6]$$

Type B(a) Formula Based on Fat in Cheese (FC)

From Formula [2], $YFFDC = C \cdot K_c + CWS + CS$. Because $CWS = WSFFDC \cdot YFFDC$ and $CS = SFFDC \cdot YFFDC$,

$$\text{then } YFFDC = C \cdot K_c + (WSFFDC + SFFDC) \cdot YFFDC = \frac{C \cdot K_c}{1 - SFFDC - WSFFDC}.$$

$$\text{Since } SFFDC = \frac{SFFC}{1 - MFFC},\ 1 - MFFC = 1 - \frac{M}{1 - FC} = \frac{1 - FC - M}{1 - FC},$$

$$\text{and } SFFC = SC/(1 - FC)$$

$$= \frac{SC/(1 - FC)}{(1 - FC - M)/(1 - FC)} = \frac{SC}{1 - FC - M},$$

And similarly,

$$WSFFDC = \frac{WSC}{1 - FC - M} = \frac{\frac{M \cdot WS}{1 - WS}}{1 - FC - M}.$$

Then:

$$YFFDC = \frac{C \cdot K_c}{1 - \frac{SC + M \cdot \frac{WS}{1 - WS}}{1 - FC - M}}.$$

$$Y = \left[F \cdot K_f + \frac{C \cdot K_c}{1 - \dfrac{SC + \dfrac{M \cdot WS}{1 - WS}}{1 - FC - M}} \right] \left[\frac{1}{1 - M} \right] \quad [7]$$

Using factors from Appendix 2 and sef = .5, Formula [7] can be expressed as:

$$Y(\text{Cheddar}) = 1.476F + 1.834C \quad [33]$$

$$Y(\text{Gouda}) = 1.596F + 2.005C \quad [37]$$

Formula [7] contains a factor FC, which depends on both F and C in a predictive formula. Therefore, the equation cannot be used, as such, without going first through an iterative exercise of estimating FC, as noted in the text. In doing this, K_f, K_c, SC, M, and WS are constants in the predictive formula. In estimating FC, a preliminary estimate of FC is made, after which an estimate of Y is calculated; then FC is calculated again using $F \cdot K_f / Y$; then a new estimate of Y is made and a new FC is determined; the exercise is repeated until two consecutive cycles yield the same FC. For example, using the data for Cheddar cheese (general formula) in Appendix 2, but without FC, a preliminary estimate of FC of .35 gives a yield of 10.018 kg; using this yield a new FC of .3342 is estimated, which gives a new yield of 9.973 kg; this gives a new FC of .3357 and next a new yield of 9.977; the new yield gives an FC of .3356, which does not change on the next cycle; the yield then is 9.977kg and FC = .3356.

Type B(b) Formula Based on Fat-in-Dry Cheese

From Formula [2]:

$YDC = C \cdot K_c + CF + CS + CWS$. Because $CF + CS + CWS = (FDC + SDC + WSDC)\, YDC$, then $YDC = C \cdot K_c + (FDC + SCD + WSDC)\, YDC$

$$= \frac{C \cdot K_c}{(1 - FDC - SDC - WSDC)}.$$

Since $YFFDC = YDC\,(1 - FDC)$,

$$\text{Then } YFFDC = \frac{C \cdot K_c}{\dfrac{1 - FDC - SDC - WSDC}{1 - FDC}}.$$

Since $WSDC = WSC \left[\frac{1}{1 - M} \right]$, $WSC = \frac{M \cdot WS}{1 - WS}$, and

$$WSDC = \frac{WS \cdot M}{(1 - WS)(1 - M)}.$$

Then:

$$Y = \left[F \cdot K_f + \frac{C \cdot K_c}{\dfrac{1 - SDC - FDC - \left[\dfrac{WS}{1 - WS} \right] \left[\dfrac{M}{1 - M} \right]}{1 - FDC}} \right] \left[\frac{1}{1 - M} \right]. \quad [8]$$

This formula is similar to that of PBK in Appendix 5. The general formula type B(b) is similar to that of type B(a) in that factor FDC depends on C and F in the milk; a similar exercise derives the FDC for new F and C. Using factors from Appendix 2 and sef = .5, Formula [8] reduces also to Formulas [33] and [37] for Cheddar and Gouda cheese.

Type C Formulas

The descriptive type C formula is based on MFFC, where moisture, salt, and whey solids are combined only with the complex of para-casein and CaH_2PO_4, with moisture as moisture in fat-free cheese (MFFC):

$$Y = \text{Fat in cheese} + \left[\frac{pcCaH_2PO_4}{\text{(Fraction of } pcCaH_2PO_4 \text{ in fat-free dry cheese)}}\right]\left[\frac{1}{1 - MFFC}\right]. \qquad [9]$$

The general formula based on MFFC is:

$$Y = F \cdot K_f + YFFC \qquad [52]$$

Type C(a) Formula Based on 1/(1 – MFFC)

Since CS = SFFDC · YFFDC, and CWS = WSFFDC · YFFDC

$$= \left[\frac{MFFC \cdot WS}{(1 - MFFC)(1 - WS)}\right] YFFDC,$$

$$YFFDC = C \cdot K_c + \left[SFFDC + \left[\frac{MFFC}{1 - MFFC}\right]\left[\frac{WS}{1 - WS}\right]\right] YFFDC$$

$$= \frac{C \cdot K_c}{1 - SFFDC - \left[\frac{MFFC}{1 - MFFC}\right]\left[\frac{WS}{1 - WS}\right]}$$

$$Y = F \cdot K_f + YFFDC\,(1/(1 - MFFC))$$

$$Y = F \cdot K_f + \left[\frac{C \cdot K_c}{1 - SFFDC - \left[\frac{MFFC}{1 - MFFC}\right]\left[\frac{WS}{1 - WS}\right]}\right]\left[\frac{1}{1 - MFFC}\right]. \qquad [10]$$

Type C(b) General Formulas

Similarly,

$$Y = F \cdot K_f + \frac{C \cdot K_c}{1 - SFFC - MFFC - \frac{MFFC \cdot WS}{1 - WS}} \qquad [11]$$

Formulas [10] and [11] simplify to identical formulas for Gouda and for Cheddar cheese from data in Appendix 2 (sef = .5):

$$Y(\text{Cheddar}) = .93F + 2.632C \quad [40]$$

$$Y(\text{Gouda}) = .936F + 2.837C \quad [42]$$

Other General Formulas

Other general formulas can be derived or can be obtained by substitution. For example, a type B formula, in which the salt is present as SC, can be obtained by substituting SC/(1 – M) for SDC in Formula [8]. "Salt in moisture" is a common desirable constant in cheese; SC, SDC, and SFFC can be described as variants of SC : M; e.g., SC = M · SC/M where SC : M is a constant. Other examples are that the complex of $pcCaH_2PO_4$ can be split into separate factors of pc and CaH_2PO_4 and that a yield formula can be derived with only the variable C in the numerator: e.g., the sponge type of Formula [23] (35).

Solute-Exclusion Factor

Appendix 9 discusses the solute-exclusion factor. These derivations did not consider it. However, they can be suitably modified by exchanging WSC or M · WS/(1 – WS) by Msef · WS/(1 – WS) (see Appendix 9 for Msef), or by exchanging WSFFC or MFFC · WS/(1 – WS) by MFFCsef · WS/(1 – WS). This has been done in Tables 1 and 3.

Table 3 contains the reduced formulas for the general formulas for Cheddar and Gouda cheese. In their reduction, unknowns, Y and C, are in the denominator. An example of a reduction for Cheddar cheese for the Type A(b) Formula 5 using values in Appendix 2 is:

$$Y = \frac{.93F + 1.01813C}{1 - .017 - .37 - .37 - .5\,(.96C - .022)/Y)\,(.065/.935)}$$

$$= \frac{.93F + 1.01813C}{.587278 + (.03337C - .00073)/Y} \quad [53]$$

Cyclical iterative calculations are used in estimating yield, starting with an estimate of Y for the denominator. A difference of Y of .1 kg in the Y in the denominator makes a difference of .0014 kg or .014% in estimated yield. A decrease in C of .1 kg/100 kg of milk only in the denominator resulted in an increase in estimated yield of .0056 kg or .056%; the same decrease in C of .1 kg throughout the formula resulted in a decrease in estimated yield of .168 kg or 1.7%. A single cycle in determining Y in the denominator from an estimate is sufficient for most purposes. In Table 3, Y was incorporated in the reduction using the Y calculated by the full formula.

APPENDIX 4

$C \cdot K_c$

Several formulas use the term "K_c", which represents a proportionality factor for the fraction of milk casein retained in the curd. It depends also on losses of casein in curd fines, retention of calcium and phosphate in the complex of para-casein and CaH_2PO_4, and losses of GMP through the action of chymosin.

K_c can be represented as:

$$K_c = \mu - pcCL/\mu \cdot C + CaH_2PO_4/pc \quad [54]$$

where:

K_c = conversion factor of casein in milk to the complex of para-casein in CaH_2PO_4 in cheese,

μ = fraction of para-casein in casein (= .96), pcCL = para-casein in curd fines (kg/100 kg of cheese milk),

$\frac{CaH_2PO_4}{pc}$ = fraction or proportion of inorganic CaH_2PO_4 to para-casein in cheese,

= [Ca in cheese + [P in cheese − (para-casein in cheese) · (proportion of P in pc)] · (MWH_2PO_4/MWP)]/pc, where MWH_2PO_4 and MWP are the molecular weights of H_2PO_4 (M_r = 97) and P (M_r = 31).

Factor pcCL/(μ · C) is not completely satisfactory for a general formula because C varies; however, for some applications an average value for C can be used. The molecular weight of H_2PO_4 was used because this is the predominant form of phosphate at pH 5.0.

The yield of para-casein from casein was taken at 96% in this paper (μ = .96). This is based on a reported proportion of 25% of GMP in κ-casein representing 16% of the total casein (21, 32). Para-casein is then .96 of casein. Karman et al. (23) represented para-casein as 95.6% of casein.

Before estimating K_c, amounts of para-casein and inorganic CaH_2PO_4 in cheese must be estimated. Para-casein is estimated by subtracting the contents of moisture, fat, salt, whey solids, and inorganic CaH_2PO_4 in cheese from 1.000. Using data in Appendix 2, the para-casein contents of Cheddar and Gouda cheeses are .2356 and .2352 where sef = 0, and .2389 and .2377 where sef = .5.

Curd losses can and do vary, but in predictive formulas for target yields, they should be the best attainable. In this paper, curd losses (pcCL) were those of PBK, viz., .022 kg of para-casein/100 kg of milk. The pcCL/(μ · C) were estimated as .00930 and .00897 for Cheddar and Gouda cheeses. The pcCL could be left in the formulas as a separate factor; C · K_c would become (C · K_{cnew} − pcCL), which is closer to the formula of PBK.

Posthumus et al. (42) introduced CaH_2PO_4 as an important element in cheese yield as part of the complex of para-casein and calcium phosphate. The contents of Ca and P in cheese varieties vary. For example, the Ca and P content of Cheddar cheese is reported as 721 and 512 mg and of Gouda cheese as 920 and 520 mg/100 kg of cheese (17). Assuming that para-casein contains .9% P (21) and that Cheddar and Gouda cheeses contain .2356 and .2352 of para-casein, their contents of CaH_2PO_4 are 1.66 and 1.89%. This is a significant part of cheese.

However, Ca and inorganic PO_4 in the whey portion of cheese would be counted twice, once in the whey solids and again in the CaH_2PO_4 fraction (Emmons and Maubois, unpublished). Calcium and P could be determined directly in the whey, which is used in the formula. They could also be estimated from the concentrations of Ca and inorganic P in cheese and in milk and from the amounts of cheese and whey produced, which was done here. Using the above levels in cheese and 119 mg Ca and 89.6 mg P (67.4 mg inorganic P)/100 g of milk (17), and yields of 9.9 g of cheese and 90.1 g of whey/100 g of milk, concentrations in Cheddar whey can be estimated as 52 mg of Ca and 43 mg of inorganic P/100 g whey for a total of .19% CaH_2PO_4 in whey. Calcium and P in Gouda whey were estimated as 65% (.042/.065) of the calculated values, because whey was diluted about .065 to .042; yields were 10.8 g of cheese and 89.2 g of whey. Estimates for whey were 21 mg Ca and 37 mg P/100 g whey for a total of .14% of CaH_2PO_4 in whey. A solute-exclusion factor of .5 was used, although it is likely less than .5 for Ca and H_2PO_4, which are smaller molecules than lactose (53). The whey-associated CaH_2PO_4 represented 3.5 and 1.5% of the total CaH_2PO_4 in Cheddar and Gouda cheese. It would be higher for cheese varieties such as Camembert and cottage (Emmons and Maubois, unpublished data). Concentrations of Ca and H_2PO_4 in cheese, higher than those in whey, were estimated as .707 and .904, totaling 1.611 g/100 g for Cheddar and .916 and .941, totaling 1.857g/100 g for Gouda. CaH_2PO_4/pc, then, are .06858 and .07895 were sef = 0 and .06729 and .07812 where sef = .5. These values were obtained by setting or estimating first M, WSC, SC, then estimating, in order, CaH_2PO_4, CaH_2PO_4/pc (using a preliminary estimate of pc), K_c (using Formula [50]), Y, FC ($F \cdot K_f / Y$), and pc, followed by a new cycle, going from the CaH_2PO_4 to pc, until there was no change.

The K_c for Cheddar and Gouda cheeses were estimated as 1.01908 and 1.03038 where sef = 0 and as 1.01813 and 1.02956 where sef = .5. It should be noted that literature values for the content of Ca and P in a cheese variety varies as well as the content of Ca and P between varieties (17).

APPENDIX 5

Formula of Van Slyke and Price

$$Y = \frac{(.93F + C - .1)\ 1.09}{1 - M} \qquad [12]$$

The formula of Van Slyke and Price (51) was originally described by Van Slyke and Publow (52); it assumes 1) that 93% of the fat in milk is recovered in cheese, 2) that .1 kg of casein/100 kg of milk is lost as curd fines and "soluble" casein, 3) that salt and whey solids in cheese equate to 9% of the fat and para-casein in the cheese, and 4) that the cheese contains, or is targeted to contain, a constant moisture content. Thus, it is a type A formula. If .1 equals 4% of the casein, then:

$$Y = \frac{(.93F + .96C)\ 1.09}{1 - M}. \qquad [56]$$

The 4% is approximately equal to the glycomacropeptide lost by action of chymosin (See Appendix 4). The formula can be further reduced, if M = .37, to:

$$Y = 1.609F + 1.661C. \qquad [24]$$

APPENDIX 6

Formula of Posthumus et al.

The General Formula. The formula of Posthumus et al. (42) is a complex formula. It is similar to the VSP formula in that there is a fat recovery factor (K_f), and moisture is portioned evenly between fat and fat-free dry cheese. It differs in that 1) salt and whey solids are related to the casein, 2) salt and whey solids are entered in the formula as separate factors, 3) casein is estimated from protein content as a proportion "λ", which varies seasonally, and 4) casein is treated not simply as casein but as a complex of para-casein and CaH_2PO_4 ($pcCaH_2PO_4$). The factors are changed slightly to conform to those in Appendix 1. It is a type B(b) formula in which FDC is used.

$$Y = \left[F \cdot K_f + \frac{\alpha \cdot P - .022}{\dfrac{1 - SDC - FDC - \left[\dfrac{WS}{1 - WS}\right]\left[\dfrac{M}{1 - M}\right]}{1 - FDC}}\right]\left[\frac{1}{1 - M}\right] \qquad [14]$$

The large denominator is called "A" (42) and is a method of converting the $pcCaH_2PO_4$ into fat-free dry cheese, taking into account the levels of solids in whey and salt in the cheese.

α. By definition (42), α is the fraction in milk, relative to protein, of para-casein and of calcium phosphate that can go to cheese. Thus:

$$\alpha = \lambda \cdot \upsilon \cdot \mu \qquad [57]$$

where λ = C/P, μ = para-casein/C, and $\upsilon = \dfrac{pc + CaH_2PO_4}{pc} = 1 + CaH_2PO_4/pc$. The λ and μ are known; CaH_2PO_4/pc was estimated in Appendix 4 as .06729 and .07812 for Cheddar and Gouda cheeses, where sef = .5.

Using data in Appendix 2, α for Cheddar and Gouda cheeses were estimated as .7889 and .8063. The .8063 for Gouda is close to that determined in commercial practice of .804 (42).

The relation of $pcCaH_2PO_4$ to α is obvious. The relation of $pcCaH_2PO_4$ and α to K_c is that the latter includes casein losses in curd fines.

It is useful to look at estimates of para-casein in the protein in milk from the PBK data. Para-casein can be estimated directly from the fractions of casein in milk, λ = .779, and of para-casein in casein, μ = .96, as .7478. Another way, if "α" (.804) contains 7.25% (1.857 (23.77 + 1.857)) of inorganic CaH_2PO_4, then the para-casein content of milk protein is .7457; this is close to the given value of .7478.

Simpler Formulas for Gouda and Cheddar Cheese. To simplify formulas for Gouda and Cheddar cheese, the factor .022 (pcCL) for the para-casein in the curd fines can be reduced to .00641P (.022/3.431) or .00832C for Gouda and to .00688P or .00893C for Cheddar cheese. Simpler formulas for cheese of constant composition using data in Appendix 2 (sef = 0) are:

$$Y(\text{Gouda}) = 1.596F + 1.582P, \alpha = .804 \quad [58]$$

$$= 1.596F + 2.031C \quad [34]$$

$$Y(\text{Cheddar}) = 1.476F + 1.452P, \alpha = .7889 \quad [59]$$

$$= 1.476F + 1.886C \quad [30]$$

APPENDIX 7

Formula of Lolkema

General Considerations. Lolkema (30) presented a somewhat different group of formulas for Gouda cheese. They are related to each other; we put some of them together into one general formula. In practice, it is convenient to keep them separate because they relate to different phases of control of the cheese operation.

$$Y = (P \cdot R_{ffdc} + F \cdot K_f) \cdot \left[\frac{1}{1 - M}\right] \quad [60]$$

where

$$R_{ffdc} = FFDC \cdot Y/P$$

$$= (.2594)(10.841/3.431)$$

$$= .9334 \text{ (from Appendix 2)}$$

$$= \frac{K_pP}{P} + \frac{.140P}{P} + \frac{CS}{P} \quad [61] \text{ [from Lolkema (30)]}$$

$$= 1.140K_p + .0742 = .9328 \text{ [typical value (30)]}$$

where .140 varies depending on quantity of water used during heating, on water content of the cheese variety, and on some other factors (30),

$$\text{where } \frac{CS}{P} = \left[\frac{SC}{P}\right]\left[\frac{Y}{100}\right]$$

$$\text{e.g., } \left[\frac{2.35}{3.431}\right]\left[\frac{10.83}{100}\right] = .0742$$

and where

$$K_p = \frac{P - K_w \cdot P_w}{P}$$

where P_w = protein content of first whey, K_w = proportion of whey to milk, e.g., .89, and K_p = e.g., .7532 (30). These factors, such as 1.140 and .0742, depend on historical data obtained from many vats. Thus, from Formula [61]:

$$R_{ffdc} = 1.140\left[\frac{P - K_w \cdot P_w}{P}\right] + .0742 \quad [62]$$

It is quite difficult to include Y, an unknown, in the "CS/P" factor; therefore, .0742 is used for Gouda and .0525 [(9.885 · 1.7)/[(3.2 · 100)] for Cheddar cheese. The general formula for Gouda cheese is, then, from Formulas [60] and [62]:

$$Y(\text{Gouda}) = \left[P + \left[1.140\frac{(P - K_w \cdot P_w)}{P} + .0742\right] + F \cdot K_f\right]\left[\frac{1}{1 - M}\right]$$

$$= F \cdot K_f + 1.2142P - 1.140\,K_w \cdot P_w)\left[\frac{1}{1 - M}\right] \quad [15]$$

It is possible to reduce this equation further, using data in Appendix 2, and where $k_w \cdot P_w$ = .849 = .24747P, to:

$$Y(\text{Gouda}) = (.9362F + .9321P)\left[\frac{1}{1 - M}\right]$$

$$= 1.596F + 1.589P \quad [63]$$

Formula for Gouda Cheese Base on Casein. It is useful to convert the formula to one based on casein for comparison to other formulas. Assuming that C = .779P (Appendix

2), Formula [63] becomes:

$$Y\ (\text{Gouda}) = 1.596F + 2.040C \qquad [35]$$

Cheddar Formula Based on Protein or Casein. Lolkema (30) did not give the factors for yield of Cheddar cheese using his formulas. It was estimated earlier that a factor for salt could be .0525 in Formula [61]. But the question remained of how to determine the Cheddar equivalent of 1.140. A factor could be derived by (a) first calculating a factor for Gouda based on PBK data through equating the "fat-free-dry-cheese" parts of Formulas [14] and [15] to obtain a PBK-modified "1.140", (b) obtaining the difference between 1.140 and the PBK-modified factor, (c) equating those parts of the equations using the PBK Formula [14] for Cheddar to obtain a factor for the Lolkema Formula [15], but based on PBK data and (d) adding to that factor in (c) the difference in (b) to obtain a so-called Cheddar factor instead of 1.140. It is estimated as follows:

(a) Let X = PBK-modified 1.140. Then:

$$X\,(P - K_w \cdot P_w) + .0742P = \frac{(\alpha \cdot P - .022)\,(1 - FDC)}{1 - SDC - FDC - \left[\dfrac{WS \cdot M}{(1 - WS)\,(1 - M)}\right]} \qquad [64]$$

Using data in Appendix 2, X (Gouda) = 1.137. (b) The difference between 1.140 and 1.137 = .003. (c) Using Formula [50], P_w for Cheddar is estimated as .8875. Using data in Appendix 2 for Cheddar (sef = 0), α = .7899, and K_w = .90 with Formula [64], then X (Cheddar) = 1.153. (d) The estimated Lolkema factor for Cheddar = 1.153 + .003 = 1.156. The estimated Lolkema formula, modified for Cheddar and based on protein, then becomes:

$$Y\ (\text{Cheddar}) = [1.156\,(P - K_w \cdot P_w) + .0525\,P) + (F \cdot K_f)]\left[\frac{1}{(1 - M)}\right]. \qquad [65]$$

Using these data and $K_w \cdot P_w$ = .9 · .8875 = .79875, Formula [65] becomes

$$Y\ (\text{Cheddar}) = 1.476 + 1.460P. \qquad [66]$$

Based on casein, where P = C/.77,

$$Y\ (\text{Cheddar}) = 1.476F + 1.896C. \qquad [31]$$

The yield in Table 3 of 9.985 kg is close to that of the general formula (sef = 0) of 9.977 kg.

Comparison of Formulas of Lolkema and Posthumus et al. The estimated yields of Gouda cheese in Table 3 by formulas of Lolkema (30) and Posthumus et al. (42) were close at 10.831 and 10.807.

The PBK formula uses separate factors for whey solids, salt, and for CaH_2PO_4 in para-casein complex, whereas protein-free whey solids, milk salts, and salt are lumped together by Lolkema (30). In practice, the α of PBK and the constants of 1.140 and .0742 of Lolkema are determined from analyses of cheese, milk, and whey.

The factor for converting milk protein to para-casein in the PBK formula was estimated in Appendix 6 to be .7478 or .7457. In the Lolkema (30) formula, $(P - K_w \cdot P_w)$ consists, by definition, of para-casein plus residual whey proteins from the whey in cheese. The content of whey solids in Gouda cheese is .0185, of which about 13% would be protein (22); this is .026 kg of whey protein in 10.831 kg of cheese, or .76% of the 3.431 kg of milk protein/100 kg of milk. The residue of para-casein of .7456 (.7532 − .0076) is slightly less than the .7478 or .7457 above.

APPENDIX 8

Modified Formula of Van Slyke and Price (15) Based on Moisture in Fat-Free Cheese

The general formula based on MFFC is:

$$Y = F \cdot K_f + YFFC \qquad [52]$$

$$Y = F \cdot K_f + \frac{C \cdot K_{ffdc}}{1 - MFFC} \qquad [67]$$

where K_{ffdc} = conversion factor of fat-free dry cheese solids in relation to casein in milk. This reformulation of the VSP formula can only be done by equating Formulas [58] and [12] on milk of a constant composition. That milk was the average milk for Cheddar in Appendix 2, containing 3.6% fat and 2.464% casein; moisture was 37% and MFFC was .5595, that calculated in Appendix 2; using those data, K_{fldc} = 1.1682. Then the modified formula becomes:

$$Y\text{ (Cheddar)} = .93F + \frac{1.1682C}{1 - MFFC} \quad [16]$$

$$= .93F + 2.652C \quad [38]$$

APPENDIX 9

Modification of General Formulas for Solute-Excluding Moisture in Cheese

Just prior to submitting this paper, additional information came to our attention that indicates that whey solids are excluded from some of the water in cheese, that which is bound to or occluded within the para-casein micelles (23, 53, 54). Larger molecules are excluded from more para-casein-associated water, and smaller molecules from less (53); an amount of water equal to ca. 50% of para-casein appears to exclude lactose. Early work by McDowall and Dolby (38) and subsequent studies by Mocquot (39) and Davies and White (12) support this. The derivation of the Maubois and Mocquot formula (35, 36, 47) uses a solute-excluding factor of .5 for the level of casein in the cheese.

The general formulas in Table 1 were modified to account for an average sef for para-casein-associated water of .5. This average sef ignores differences in sef for the constituents of whey – salt, lactose, and whey proteins. The term M in the general formulas, which is associated with the whey solids, is substituted with:

$$Msef = M - sef\,(\mu \cdot C - pcCL)Y; \quad [68]$$

MFFC becomes MFFCsef =

$$MFFC - (sef\,(\mu \cdot C - pcCL)/Y)/(1 - FC) \quad [69]$$

Yields for Cheddar and Gouda cheese were estimated for the four modified general formulas as in Table 1. All the calculations used the cyclic iterative calculations (Appendix 3) because of the unknown Y, FC, or FDC in the formulas. The estimated yields were 1.5% less for Cheddar and 1.0% less for Gouda cheese at 9.834 and 10.736 kg as compared with 9.977 and 10.842 kg (Table 3); these were less than those estimated by other formulas, also less than those obtained in commercial practice (30, 42), and slightly less than that obtained by the VSP formula (51). The finding by Karman et al. (23) that some of the proteose-peptose is retained in rennet curd may apply here. These observations emphasize further the necessity of detailed experiments on retention of components in cheese and on verification of yield formulas.