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## RESEARCH PAPERS

### Estimating Values of Milk Components to a Dairy Manufacturer<sup>1</sup>

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#### ABSTRACT

A method is developed for determining money values per point of protein and fat in a processing plant. The value per point of solids is computed from the effect of variation in solids test on the firm's profits. It can be extended to apply to solids-not-fat instead of protein and to various manufactured products. The procedure is presented in worksheet form and applied to a cheese plant. In the plant studied, the maximum premium per point of protein per 100 kg of milk is \$.2891.

#### INTRODUCTION

Incorporation of a payment for protein or solids-not-fat into a multiple component milk pricing system has been considered for a number of years. Arguments in favor of a two component formula have been concerned primarily with two main issues: producer payment equity and consumer preferences.

The equity question involves the desirability of paying producers relative to some true value of their milk. Payment based strictly on fat content ignores commercial value of protein or solids-not-fat content. Producers with low ratios of protein to fat solids in their milk receive a disproportionately large share of the total milk payments.

Consumers prefer higher protein or solids-not-fat content in fluid milk at all percents of fat. Consumption of fats has become increasingly unpopular with a weight- and health-conscious

public. Payment for milk based on fat content alone ignores these expressed preferences and encourages producers to take an opposite position in developing their product.

Adoption of a multiple component pricing formula on a nationwide basis would help to correct these problems. No national movement is necessary, however, for individual firms to adopt this type of pricing scheme, and some have chosen to.

A number of studies have dealt with multiple-component pricing methods: Ahlgran (1), Brog (2, 3), Clarke and Hassler (4), Froker and Hardin (5), Hillers (6), Johnson (7), and Luke (8) among others. Some studies have dealt with the problem a firm faces in implementing a multiple-component pricing scheme: how to determine the price paid for each component.

Studies of values of milk components typically have been based on value of marginal product of the milk components. The value of additional milk product that is produced when a component test is increased is interpreted as the value of that additional component. Methods for strict value of marginal product assume that the increased quantity of product costs nothing to produce. This additional product does have associated costs, and to ignore these costs leads to component values that are excessively high. In most of these valuations, an increase in one component is accompanied by an increase in other components by the components' standard content relationship. But assignment of the full value of the marginal product to one component when other components also are increased ignores the contribution of the other components.

Some studies of component pricing have concerned a more aggregative issue than the one facing an individual firm. Given a total pool of milk and a total pool of receipts from milk buyers for that milk, they consider price differentials for distributing total receipts among producers. This issue is not independent

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of the first. The formula adopted for distributing receipts among producers should be consistent with values of components to milk buyers.

This paper considers the effect on plant profits of an increase in protein test in milk. In considering firm profits, effects of both revenue and cost of the change in protein test enter into the determination of protein value. A general framework is developed first; then an example is outlined in a worksheet form.

#### DEVELOPMENT OF PROCEDURE

A brief outline of the development of a procedure for measuring protein premium is presented first. The first step is to consider the firm's revenues, costs, and profits (= revenues - cost). The next step is to consider the effect upon the firm's profits of variation in its total receipts of protein. This effect is converted into a measure of the effect upon profits of variation in protein test. This last effect is used to determine the maximum premium the firm can afford to pay for milk that has a higher protein test without reducing its profits. This paper studies effect of variation in protein receipts upon a firm's revenues, costs, and profits. Because differential calculus provides a method for studying effects of variation in one variable upon related variables, it is used in this paper. In addition, application of differential calculus to a relation results in a complete accounting for all effects of a change in one variable. Its use, therefore, reduces the chance of overlooking significant effects. Another reason is proper assignment of effects to causes. Suppose, for example, that variation in causal forces one and two affects dependent variables three, four, and five. Differential calculus determines the total change in each of the last three variables and also determines how much of each total change is caused by variation in variable one and how much is caused by variation in two.

Let  $q$  represent the total amount of cheddar

cheese produced and  $v$  represent the total volume of milk used in cheddar cheese. The  $q$  depends upon  $v$ , which can be expressed as  $q = Q(v)$ . But the amount of cheese produced from  $v$  depends upon the amounts of protein, fat, and water in the milk. Let the total amounts of fat, protein, and water and other materials in the milk be  $f$ ,  $p$ , and  $w$ . For short,  $w$  will be referred to simply as "water". Then<sup>3</sup>

$$q = Q(f, p, w) \quad [1]$$

where  $v = f + p + w$ . If the selling price for cheese is  $s$ , the firm's total revenue from cheese is

$$\text{Rev} = sq = sQ(f, p, w) \quad [2]$$

The costs that the firm experiences in producing cheese can be lumped into two categories: costs of milk and costs of operation. If we let  $r$  = average price paid for milk and  $m$  = total cost of milk, then

$$m = vr = (f + p + w) r \quad [3]$$

The average price paid for milk depends upon the fat and protein test of milk received. Let  $p_t$ ,  $f_t$ , and  $w_t$  represent protein test, fat test, and "water test" of milk received so that  $f_t + p_t + w_t = 1.0$ . (This means that  $f_t$ ,  $p_t$ , and  $w_t$  are proportions, not percentages. For milk of 3% butterfat,  $f_t = .03$ .) Then  $r$  is a function of  $p_t$  and  $f_t$ , say

$$r = R(f_t, p_t) = R(f/v, p/v) \quad [4]$$

and milk cost can be written

$$m = (f + p + w) R(f_t, p_t) \quad [5]$$

The operating costs include all costs other than milk costs. Letting  $c$  = total operating costs and  $a$  = average operating costs (i.e., operating cost per kilogram of cheese produced), then

$$c = aq \quad [6]$$

Average operating cost varies as the volume and composition of milk received varies so we can express  $c$  as a function of  $f$ ,  $p$ , and  $w$ , say

$$a = A(f, p, w) \quad [7]$$

<sup>3</sup>In this paper a lower case letter symbolizes a variable. The corresponding upper case letter symbolizes the function that determines the value of the variable. Thus  $q$  represents the amount of cheese produced and  $Q(f, p, w)$  is a function that determines the amount of cheese produced. The letters  $f$ ,  $p$ , and  $w$  represent variables that affect the value of  $q$ .

and

$$c = A(f, p, w) Q(f, p, w) \quad [8]$$

The firm's profit ( $\pi$ ) is its excess of total revenues over total costs. From [2], [5], and [8], this can be written

$$\pi = sQ(f, p, w) - A(f, p, w) Q(f, p, w) - (f + p + w) R(f, p, w) \quad [9]$$

Expression [9] shows that  $\pi$  is a function of  $f$ ,  $p$ , and  $w$ . From here on, the procedure is: a) Determine how  $\pi$  is affected by variation in  $f$ ,  $p$ , and  $w$ , b) fix the volume of milk received ( $f + p + w$ ) and vary protein receipts (and consequently protein test), c) vary receipts of water to offset variation in protein receipts, d) use results of step a) to determine effect of steps b) and c) on profits, and e) use results of step d) to set an upper limit to price premium a firm can afford to pay for additional protein.

Taking the total differential of [9] with respect to  $f$ ,  $p$ , and  $w$  shows the effect on profit of variation in  $f$ ,  $p$ , and  $w$ . Letting  $dp$ ,  $df$ , and  $dw$  be the changes in  $p$ ,  $f$ , and  $w$ , the total differential of [9] is<sup>4</sup>

$$d\pi = (s \partial Q/\partial p - a \partial Q/\partial p - q \partial A/\partial p - v \partial R/\partial p - r) dp + (s \partial Q/\partial f - a \partial Q/\partial f - q \partial A/\partial f - v \partial R/\partial f - r) df + (s \partial Q/\partial w - a \partial Q/\partial w - q \partial A/\partial w - v \partial R/\partial w - r) dw \quad [10]$$

Equation [10] will be manipulated algebraically to derive component premiums.

Before deriving premiums, let us pause to interpret [10] in terms that are relevant to a cheese plant. Profit equals revenues minus costs, and the change in profit equals change in revenues minus change in costs. The term  $\partial Q/\partial p$  is the marginal physical product of protein. It equals the change in cheese output per unit change in amount of protein used;  $dp$  equals the change in amount of protein used.

The term  $(\partial Q/\partial p) dp$ , therefore, equals the change in cheese output resulting from the use of  $dp$  additional protein. The sum  $(\partial Q/\partial p) dp + (\partial Q/\partial f) df + (\partial Q/\partial w) dw$  is the total differential of the production function (1). Therefore, we can write

$$s [(\partial Q/\partial p) dp + (\partial Q/\partial f) df + (\partial Q/\partial w) dw] = sdQ$$

The  $dQ$  is total change in cheese output and  $s$  is price of cheese so  $sdQ$  is revenue from the additional cheese. The remaining terms in [10] are subtracted from  $sdQ$  and represent changes in costs

$$a [(\partial Q/\partial p) dp + (\partial Q/\partial f) df + (\partial Q/\partial w) dw] = adQ$$

The variable  $a$  is average operating cost. The product  $adQ$  is the amount it would cost to produce the additional cheese. Increasing output tends to reduce average operating cost. The term

$$[(\partial A/\partial p) dp + (\partial A/\partial f) df + (\partial A/\partial w) dw]$$

is the total differential of the average cost function  $a = A(f, p, w)$  and represents the change in average operating cost that results from increasing cheese output. Thus,

$$q [(\partial A/\partial p) dp + (\partial A/\partial f) df + (\partial A/\partial w) dw] = qdA$$

represents the change in the cost of producing the original output of cheese.

The term  $\partial R/\partial p$  is the change in price paid for milk that results from a one unit increase in protein receipts (and a resulting change in protein test).

$$v [(\partial R/\partial p) dp + (\partial R/\partial f) df + (\partial R/\partial w) dw] = vdR$$

The  $dR$  is the change in milk price that results from variation in  $p$ ,  $f$ , and  $w$ ;  $vdR$  is the resulting

<sup>4</sup>In the rest of the paper a notational shorthand will be used in the partial derivatives. The capital letter that identifies a function will be used alone. For example,  $\partial Q(f, p, w)/\partial p$  will be shortened to  $\partial Q/\partial p$  and  $\partial A(f, p, w)/\partial w$  will be shortened to  $\partial A/\partial w$ .

change in the cost of obtaining the original volume of milk. Finally,

$$r(dp + df + dw) - r dv$$

is the cost of the additional milk.

A protein premium is the difference between the prices paid at plant for two deliveries of milk that have the same fat test, but one delivery has a one-point higher protein test (and a lower water test) than the other. To determine a protein premium, therefore, set  $dp > 0$ ,  $dw = -dp$ , and  $df = 0$ . It is convenient to choose the units of measurement so that  $dp = 1$  and  $dw = -1$ . The firm's profits are not to be reduced by the increased protein receipts and the resulting higher outlays on milk, that is  $d\pi > 0$ . Then [10] can be rewritten as

$$\begin{aligned} d\pi &= (s-a)(\partial Q/\partial p - \partial Q/\partial w) \\ &\quad - q(\partial A/\partial p - \partial A/\partial w) \\ &\quad - v(\partial R/\partial p - \partial R/\partial w) > 0 \end{aligned} \quad [11]$$

The difference  $\partial R/\partial p - \partial R/\partial w$  is protein differential, the increase in milk price for an increase in protein test with an equal reduction in water-test. For brevity, call it  $dR/dp$ . Solving [11] for  $dR/dp$  yields [12].

$$\begin{aligned} dR/dp &< [(s-a)(\partial Q/\partial p - \partial Q/\partial w) \\ &\quad - q(\partial A/\partial p - \partial A/\partial w)]/v \end{aligned} \quad [12]$$

If  $dR/dp$  is smaller than the right side, the protein premium is small enough that profits are increased by increased protein receipts. If  $dR/dp$  were to exceed the right side of [12], the protein premium would be so large that increased protein receipts would reduce profits. If  $dR/dp$  equals the right side, the protein premium is such that profits are neither increased nor reduced by the increased protein receipts. Because we already have specified that the firm's profits are not to be reduced, the maximum value of  $dR/dp$  is

$$\begin{aligned} \max dR/dp &= [(s-a)(\partial Q/\partial p - \partial Q/\partial w) \\ &\quad - q(\partial A/\partial p - \partial A/\partial w)]/v \end{aligned} \quad [13]$$

Because  $dp = 1 = -dw$  and  $df = 0$ , this can be rewritten as

$$\max dR/dp = [(s-a) dQ - qdA]/v \quad [14]$$

To use expressions [12], [13], or [14] a firm needs to know  $s$ ,  $a$ ,  $q$ , and  $v$ , cheese price, average operating cost, volume of cheese produced, and volume of milk received. It then needs to determine the variation in cheese output from changing protein and water receipts ( $\partial Q/\partial p - \partial Q/\partial w$ ) and the change in average operating costs ( $\partial A/\partial p - \partial A/\partial w$ ).

Application of expression [14] may yield different  $dR/dp$  for different cheese plants, or even for the same cheese plant at different times. The  $dR/dp$  is affected by cheese price and average operating cost, which is affected by wage and utilities rates, volume of production, capacity of plant, and technology.

#### APPLICATION OF PROCEDURE

The approach represented in expression [14] can be represented conveniently in a worksheet. One such worksheet is in Table 1, which has been worked out for the cheddar cheese plant discussed in Tracy (9).<sup>5</sup>

This plant has a weekly capacity of 362,874 kg of milk. The plant is assumed to be producing cheddar cheese that contains 40% water. Twenty-two workers, including supervisory personnel, are needed to operate the plant. Whey is sold at a price that equals its handling cost. The plant typically receives milk that tests 3.60% fat and 3.05% protein. At some point, the protein test is increased by one point to 3.15%.

Letting  $c_w$  = proportion of water in cheese, the plant's output is determined from

$$\begin{aligned} \text{kg cheese per 100 kg milk} &= \\ &= (93 f_t + 77 p_t - .1)(1.09)/(1 - c_w) \end{aligned} \quad [15]$$

where  $f_t$  and  $p_t$  are proportions, not percent. (Thus  $f_t = .036$ ,  $p_t = .0305$ , and  $c_w = .4$ .) Substituting these into [15] yields 10.1670 kg cheese/100 kg milk. Then  $q$  = total cheese

<sup>5</sup>In this example, the computations will be carried to more decimal places than would be done in a practical application.

TABLE 1. Worksheet for computing protein differential.

Section	Item and line	Situation A (original)	Situation B
I. Basic information	1. Milk receipts (100 kg)	3,628.74	3,628.74
	2. Cheese price (\$/kg)	2.315	2.315
	3. Fat test (%)	3.6	3.6
	4. Protein test (%)	3.05	3.15
	5. Cheese output (kg)	36,893.4	37,401.1
	6. Increase in protein receipts (100 kg), (.001) × (1A)	...	3,628.74
	7. Increase in cheese output (kg), (5B) - (5A)	...	507.7
II. Revenue	8. Revenue from increased output (\$), (2B) × (7B)	...	1,175.33
III. Energy costs	9. Total (\$)	337.01	341.33
	10. Average (\$/kg of cheese), (10) ÷ (5)	.00914	.00913
	11. Change in average (\$/kg), (10B) - (10A)		-.00001
IV. Labor costs	12. Total (\$)	8,800	8,866.05
	13. Avg (\$/kg of cheese), (12) ÷ (5)	.23852	.23705
	14. Change in average (\$/kg), (13B) - (13A)		-.00147
V. Nonmilk materials costs	15. Total (\$)	4,066.76	4,122.72
	16. Avg (\$/kg of cheese), (16) ÷ (5)	.11023	.11023
	17. Change in average (\$/kg), (16B) - (16A)		0
VI. Plant, equipment, marketing, administration costs	18. Total (\$)	15,437.28	15,437.28
	19. Avg (\$/kg of cheese), (8) ÷ (5)	.41843	.41275
	20. Change in average (\$/kg of cheese), (19B) - (19A)		-.00568
VII. Total operating costs	21. Avg (\$/kg of cheese), (10) + (13) + (16) + (19)	.77632	.76916
	22. Total cost of increased output (\$), (7B) × (21B)		390.50253
	23. Profit from increased output (\$), (8B) - (22B)		784.83
VIII. Protein differential	24. Change in avg cost (\$/kg), (21B) - (21A)		-.00716
	25. Change in total cost of original output (\$), (5A) × (24B)		-264.16
	26. Total available for protein (\$), (23B) - (25B)		1,048.99
	27. Maximum premium per point (\$/100 kg of milk), (26B) ÷ (1B)		.2891

output =  $10.1670 \times 3628.74 = 36,893.4$  kg.  
This basic information is entered in section I of the worksheet. By equation [15], the production function for this particular firm, corresponding to equation [1], can be written as

$$q = Q(f, p, w) \quad [16]$$

$$= (f + p + w)(93 f_t + 77 p_t - .1)(1.09)/1 - c_w)$$

It is necessary to determine the increase in cheese production that results from increasing  $p_t$  to .0315 and reducing  $w_t$ . This can be done by evaluating  $dQ$  (which equals  $\partial Q/\partial p - \partial Q/\partial w$  from [16]). A more direct procedure is to change the value of  $p_t$  in [15] from .0305 to

.0315. The result is 10.3069 kg cheese/100 kg milk. Because  $dp = -dw$ , total milk receipts are constant. The new cheese output is, therefore,  $10.3069 \times 3628.74 = 37,401.1$  kg. This is entered in line 5B, and the difference between the entries on lines 5B and 5A is the increase in volume of cheese produced, and this is entered on line 7B. The entry on line 7B is  $dQ$ .

The firm's operating costs are divided into four categories: energy; labor; nonmilk materials; and costs of plant, equipment, marketing, and administration. The totals for these are entered on lines 9A, 12A, 15A, and 18A. The costs per kilogram of cheese for the categories are entered on lines 10A, 13A, 16A, and 19A and their sum is entered on line 21A.

The next step is to compute the change in the average cost,  $dA$ . The primary impact of the increased protein test on energy costs is to reduce the amount of steam required for heating milk. The one-point increase in protein test will lower the steam requirement by about .22 kg/100 kg of milk. This results in a reduction in the cost per kilogram of cheese of  $-.001\text{¢/kg}$  (entered on line 11B).

Part IV of the worksheet concerns labor costs. The higher protein content affects only the number of hours worked by those workers handling the additional cheese. If the present work force can handle the additional cheese, total labor cost will remain the same. In this worksheet, workers handling cheese are employed fully in the initial situation. The 1.3761% increase in cheese output requires a 1.3761% increase in the number of hours worked by these employees, and these additional hours are all overtime. As a result, total labor cost rises by \$66.0525 and the new total labor cost is entered on line 12B. The new average labor cost is entered on line 13B.

Nonmilk materials include salt, color, preservatives, and packaging materials. Each kilogram of cheese requires about 11¢ worth of these materials. See lines 16A and 16B.

Section VI deals with so-called fixed costs, i.e., cost items whose totals do not change even though total output changes. Because the total does not change (compare 18A and 18B), it follows that average fixed cost declines. (Compare lines 19A and 19B.)

Section VII is derived from Sections III, IV, V, and VI. The cost per kilogram of cheese for the new volume of cheese is entered on line

21B. The total cost of the increased output of cheese ( $adQ$ ) is computed on line 22B. The profit from the increased output is entered on line 23B. This entry equals the value of  $(s-a)dQ$ . The change in average cost is  $-.00716$  on line 24B. This equals  $dA$ . The term  $qdA$  is computed on line 25B.

Recall that

$$\max dR/dp = [(s-a)dQ - qdA]/v$$

To obtain  $dR/dp$ , line 25B is subtracted from line 23B. The result is entered on line 26B. Then  $\max dR/dp$  is obtained by dividing the entry on 26B by total milk receipts. See line 27B.

## RESULTS

The protein premium for this cheese plant is \$.2891 per point of protein/100 kg of milk. This is equivalent to \$.1311 per point of protein/100 lb of milk.

If this plant is receiving 272,200 kg milk/week, the protein premium is \$.3072. If it is receiving 453,400 kg/week, the protein premium is \$.2897.

Ignoring the variation in average operating costs (i.e., operating cost per unit of cheese produced) that results when output of cheese increases results in an incorrect measure of maximum premium. The reason for this can be made clear from expression [14]. In Table 1, entries on lines 21A and 21B show that average operating costs decline as cheese production rises. That is,  $dA$  is negative. If we assumed (wrongly) that  $dA$  were zero, [14] could be written as, say,

$$\max' dR/dp = (s-a)dQ/v$$

Subtracting this expression from [14] yields

$$\max dR/dp - \max' dR/dp = -qdA/v$$

Now  $-qdA/v$  is positive. Therefore,  $\max dR/dp > \max' dR/dp$ . Thus if one assumes  $dA = 0$  when actually  $dA < 0$ , the maximum premium one will obtain is too small.

Assuming (incorrectly) that average operating costs were constant would amount to eliminating lines 24 and 25 from the table. Then the total available for protein premium would be \$784.83 (from line 23B), and the maximum

premium per point of protein would be  $\$784.83 \div 3,628.74 = \$.2163$ , about 7.25¢ less than the correct figure in the table.

#### EXTENSIONS OF PROCEDURE

A procedure similar to the one used earlier to find  $\max dR/dp$  in [14] can be used to find  $\max dR/df$ : a maximum fat premium. Letting  $df = 1$ ,  $dw = -1$ , and  $dp = 0$  in [10] leads to

$$\begin{aligned} \max dR/df = & \\ & [(s-a)(\partial Q/\partial f - \partial Q/\partial w) \\ & - q(\partial A/\partial f - \partial A/\partial w)]/v \end{aligned} \quad [17]$$

Suppose a firm does not wish to pay both a butterfat and protein differential but wishes only to pay a butterfat differential, but wants its butterfat differential per point of butterfat to reflect the value of additional protein that accompanies an additional point of butterfat on average. In equation [10], set  $df = 1$  and  $dp = \beta$  where  $\beta = dp/df$ : the increased protein received, on the average, per unit increase in fat. Then set  $dw = -(1 + \beta)$  so that total volume of milk is unchanged. Expression [11] then becomes

$$\begin{aligned} d\pi = & (s-a)(\partial Q/\partial f - \partial Q/\partial w) - q(\partial A/\partial f - \partial A/\partial w) \\ & + \beta [(s-a)(\partial Q/\partial p - \partial Q/\partial w) - q(\partial A/\partial p - \partial A/\partial w)] \\ & - v [(\partial R/\partial f - \partial R/\partial w) + \beta(\partial R/\partial p - \partial R/\partial w)] \geq 0 \end{aligned} \quad [18]$$

The term within brackets on the last line of [18] is the fat differential. For brevity, write it as  $dR'/df$ . Expression [18] can then be manipulated to yield

$$\begin{aligned} \max dR'/df = \max dR/df \\ + \beta \max dR/dp \end{aligned} \quad [19]$$

where  $\max dR/df$  and  $\max dR/dp$  are given by expressions [14] and [17].

Expression [19] and the argument used to derive it also make it clear why we set  $dp > 0$ ,  $dw = -dp$ , and  $df = 0$  in determining maximum protein premium in [14] and set  $df > 0$ ,  $dw = -df$ , and  $dp = 0$  in setting maximum fat differential in [17]. If a butterfat differential is determined by varying fat receipts by  $df$  and varying protein receipts by  $dp = \beta df$  (where  $\beta$  represents the average increase in protein per

unit increase in fat), the maximum butterfat differential is determined in [19]. But only part of this butterfat differential is due to an increase in butterfat test. This is the part identified as  $\max dR/df$ . Part of this butterfat differential is due to increased protein receipts. This is the part labeled  $\beta \max dR/dp$ . The butterfat differential that results from using [19] is, consequently, a solids differential. It is a payment for protein and for fat.

The preceding mathematical and numerical discussion has concerned determination of protein and fat differentials in a cheddar cheese plant. But the procedure is appropriate for solids-not-fat as well as for protein, and for other manufactured dairy products. All that needs to be done to the mathematical analysis to extend it is to define  $p$  as volume of nonfat solids,  $w$  as water (not water and other), and  $q$  as output of manufactured product. The essential sets of computations are five in number. They are: a) Determine the effect of changing solids (fat, protein, nonfat solids) receipts upon output. This effect is  $(\partial Q/\partial p - \partial Q/\partial w)$  in [13], or  $(\partial Q/\partial f - \partial A/\partial w)$  in [17], or  $dQ$  for short. b) Determine the revenue from the increased output  $s dQ$ . c) Determine the cost of producing the additional output  $adQ$ . d) Determine the change in the cost of the original output  $dqA$ . e) Combine the results of steps b), c), and d).

Some firms find it more convenient to measure average operating costs in dollars per 100 kg of milk received rather than in dollar per kilogram of product. The preceding analysis can be adapted easily to fit such firms. Instead of expressing total operating costs as  $c = aq = A(f, p, w) Q(f, p, w)$  as in expressions [6] through [8], express total operating costs as

$$c = vb = [f + p + w] b \quad [20]$$

where  $v$  is, as before, volume of milk received and  $b$  is operating cost per unit of milk received. Expression [7] is replaced by

$$b = B(f, p, w) \quad [21]$$

In the profit equation, [9],  $A(f, p, w) Q(f, p, w)$  is replaced by  $(f + p + w) B(f, p, w)$ . Expression [13] becomes

$$\max dR/dp = s(\partial Q/\partial p - \partial Q/\partial w)$$

$$-v(\partial B/\partial p - \partial B/\partial w)$$

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